Calculus Quiz 4 ENG-D

Class: ______________________
Student Number: _____________
Name: ________________

1. (5 points) Use the Mean Value Theorem to prove the inequality

\[ |\sin a - \sin b| \leq |a - b| \]

for all \(a\) and \(b\).

\[ \text{Let } f(x) = \sin x \text{ and } f\left(b\right) < f\left(a\right) \text{, Then } f\left(x\right) \text{ is continuous on } [b, a], \text{ and differentiable on } (b, a). \]

By the Mean Value Theorem, there is a number \(c \in (b, a)\) with

\[ \sin a - \sin b = f\left(c\right) \left(a - b\right) = (\cos c) \left(a - b\right) \]

\[ \Rightarrow |\sin a - \sin b| \leq |\cos c| |b - a| = |b - a| \]

\(\circ\) If \(a < b\), \( |\sin a - \sin b| = |\sin b - \sin a| \leq |b - a| = |a - b| \)

\(\bullet\) If \(a = b\), both sides of the inequality are 0.

2. (5 points) Find a cubic function

\[ f(x) = ax^3 + bx^2 + cx + d \]

that has a local maximum value of 3 at -2 and a local minimum value of 0 at 1.

\[ f(-2) = -8a + 4b - 2c + d = 3 \quad \circ \]

\[ f(1) = a + b + c + d = 0 \quad \circ \]

\[ f(x) = 3ax^2 + 2bx + c \]

\[ f(-2) = 12a - 4b + c = 0 \quad \circ\]

\[ f(1) = 3a + 2b + c = 0 \quad \circ \]

\(\circ\) - \(\circ\)

\[ -9a + 3b - 3c = 3 \]

\[ \Rightarrow -3a + b - c = 1 \quad \circ \]

\(\circ\) + \(\circ\)

\[ \begin{cases} a - 3b = 1 \\ a - 6b = 0 \end{cases} \Rightarrow \begin{cases} b = \frac{1}{3} \\ a = \frac{2}{9} \end{cases} \]

\(\circ\) - \(\circ\)

\[ f(x) = \frac{2}{9} x^3 + \frac{1}{3} x^2 - \frac{4}{3} x + \frac{2}{9} \]