Calculus Quiz 5 ENG-E

Class: ____________________________
Student Number: __________________
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1. (5 points) Find the area of the largest rectangle that can be inscribed in a circle of radius $r$.

The area of the rectangle is $4xy$.

Also $r^2 = x^2 + y^2 \Rightarrow y = \sqrt{r^2 - x^2}

\Rightarrow A(x) = 4xy = 4x\sqrt{r^2 - x^2} \quad A'(x) = \frac{4(r^2-2x^2)}{\sqrt{r^2 - x^2}}$

The critical number is $x = \frac{1}{\sqrt{2}} r \Rightarrow y = \frac{1}{\sqrt{2}} r = x$.

\therefore The area is $4xy = 2r^2$.

2. (5 points) A particle is moving with the given data. Find the position of the particle.

$a(t) = 10 \sin t + 3 \cos t, s(0) = 0, s(2\pi) = 12$

\[ a(t) = 10 \sin t + 3 \cos t = v'(t) \]

\[ v(t) = -10 \cos t + 3 \sin t + C = s'(t) \]

\[ s(t) = -10 \sin t - 3 \cos t + Ct + D \]

$s(0) = -3 + D = 0 \Rightarrow D = 3$

$s(2\pi) = -3 + 2\pi C + 3 = 12 \Rightarrow C = \frac{6}{2\pi}$

\therefore $s(t) = -10 \sin t - 3 \cos t + \frac{6}{2\pi} t + 3$