Calculus Quiz 5 SCI-C

Class:  
Student Number:  
Name:  

1. (5 points) \( y = f(x) = \frac{2x^2}{x^2 - 1} \).
   a. Is \( f(x) \) an even or odd function?
   b. Find \( \lim_{x \to \infty} f(x) \).
   c. Find \( \lim_{x \to 1^-} f(x) \).
   d. Find horizontal asymptotes and vertical asymptotes of \( y = f(x) \).

   a. \( f(-x) = \frac{2(-x)^2}{(-x)^2 - 1} = \frac{2x^2}{x^2 - 1} \)
   \( \Rightarrow f(x) \) is an even function.

   b. \( \lim_{x \to \infty} \frac{2x^2}{x^2 - 1} = \lim_{x \to \infty} \frac{2}{1 - \frac{1}{x^2}} = 2 \)

   c. \( \lim_{x \to 1^-} \frac{2x^2}{x^2 - 1} = -\infty \)
   (since the numerator is positive
   and the denominator approaches 0 from the negative side as \( x \to 1^- \))

   d. By (b) and \( \lim_{x \to \infty} \frac{2x^2}{x^2 - 1} = \lim_{x \to \infty} \frac{2}{1 - \frac{1}{x^2}} = 2 \)
   \( \Rightarrow \) the line \( y = 2 \) is a horizontal asymptote.

   By (c) and \( \lim_{x \to 1^+} \frac{2x^2}{x^2 - 1} = \infty \) and
   \( \lim_{x \to 1^-} \frac{2x^2}{x^2 - 1} = -\infty \) and \( \lim_{x \to 1^-} \frac{2x^2}{x^2 - 1} = 0 \)
   \( \Rightarrow \) the lines \( x = 1 \) and \( x = -1 \) are vertical asymptotes.

2. (5 points) Find a function \( f \) such that \( f'(x) = 8x^3 \) and the line \( x + y = 0 \) is tangent to the graph of \( f \).

   \( f'(x) = 8x^3 \)  \( \Rightarrow f(x) = 2x^4 + C \)
   \( f'(x) = 8x^3 = -1 \)
   \( \Rightarrow x = -\frac{1}{2} \)

   \(-\frac{1}{2} \) + \( y = 0 \)
   \( \Rightarrow y = \frac{1}{2} \)