

Calculus Quiz 1

1. (5 pts)

a. Find the lowest degree of polynomial $f(x)$ such that

$$\lim_{x \rightarrow i} \frac{f(x)}{x - i} = i, \quad i = 1, 2, 3.$$

b. Evaluate the limit $\lim_{n \rightarrow \infty} x^3 [\sqrt{x^2 + \sqrt{x^4 + 1}} - \sqrt{2}x]$.

2. (5 pts)

- a. Consider the Fibonacci sequence $1, 1, 2, 3, 5, 8, \dots$. That is, the n th term of the sequence satisfies the recurrence relation

$$a_n = a_{n-1} + a_{n-2}$$

Show that the limit of consecutive quotient $\frac{a_{n+1}}{a_n}$ is the

golden ratio $\frac{1 + \sqrt{5}}{2} \doteq 1.61803$.

- b. Consider the sequence with recurrence relation

$$a_{n+1} = \frac{a_n}{2} + \frac{1}{a_n}, \quad a_1 = 3$$

Show that $\lim_{n \rightarrow \infty} a_n = \sqrt{2}$.