

# Calculus Quiz 2

1. (5 pts) Find the limits  $L = \lim_{x \rightarrow a} f(x)$  for following given functions  $f(x)$  and  $a$ . And find a number  $\delta > 0$  such that for all  $x$

$$0 < |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

with given  $\varepsilon$ .

a.  $f(x) = \sqrt{1 - 5x}$ ,  $a = -3$ ,  $\varepsilon = 0.5$ .

b.  $f(x) = \frac{4}{x}$ ,  $a = 2$ ,  $\varepsilon = 0.4$ .

2. (5 pts)

- a. Let  $f(x)$  be a function defined on  $[a, b]$  and for any  $y$  between  $f(a)$  and  $f(b)$ , there is  $c \in [a, b]$  such that  $f(c) = y$ . Is it true that  $f$  is continuous on  $[a, b]$ ? If not, find a counterexample.
- b. Suppose  $f$  is a continuous function on  $[0, 1]$  and  $0 \leq f(x) \leq 1$ ,  $\forall x \in [0, 1]$ . Show that there must exist  $c \in [0, 1]$  such that  $f(c) = c$ .