Calculus Quiz 3

1. (5 pts)

a. Find the derivative of the function $g(x) = \dfrac{1}{\sqrt{x}}$ by using the definition of derivative.

b. Let f be a smooth function defined on $\mathbb{R}$ and $c \in \mathbb{R}$. If $f'(c) = a,\ f''(c) = b$. Evaluate the following limit

$$\lim_{h \to 0} \left[\frac{2f(c+h) - 4f(c) + 2f(c-h)}{3h^2} + \frac{f(c+h) - f(c-h)}{h} \right]$$

2. (5 pts)

a. Let $f(x)$ be a function satisfying $|f(x)| \leq x^2$ for $-1 \leq x \leq 1$. Show that f is differentiable at $x = 0$ and find $f'(0)$.

b. Show that

$$f(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is differentiable at $x = 0$ and find $f'(0)$.