

Calculus Quiz 5

1. (5 pts)

- a. If $F(x) = f(xf(xf(x)))$, where $f(1) = 2$, $f(2) = 3$, $f'(1) = 4$, $f'(2) = 5$, and $f'(3) = 6$. Find $F'(1)$.
- b. Find the points on the curve $xy^2 + yx^2 = 2$ where the tangent line is horizontal or vertical.

2. (5 pts)

a. Show that the n th derivative of $\cos^3 x$ is

$$\frac{1}{4} \left[3^n \cos \left(3x + \frac{n\pi}{2} \right) + 3 \cos \left(x + \frac{n\pi}{2} \right) \right]$$

b. Show that the implicit function defined by quadratic form $ax^2 + 2bxy + cy^2 + 2dx + 2ey + k = 0$ has first and second derivative as

$$\frac{dy}{dx} = -\frac{ax + by + d}{bx + cy + e}, \quad \frac{d^2y}{dx^2} = \frac{\Delta}{(bx + cy + e)^3}$$

where

$$\Delta = \begin{vmatrix} a & b & d \\ b & c & e \\ d & e & k \end{vmatrix}$$