

Calculus Quiz 10

1. (5 pts)

a. Let $F(x) = \int_1^x \int_{\sqrt{t}}^{t^2} \frac{\sqrt{1+u^4}}{u} du dt$. Find $F''(2)$.

b. Find a function f and a number a such that

$$6 + \int_a^x \frac{f(t)}{t^2} dt = 2\sqrt{x}, \text{ for all } x > 0$$

2. (5 pts)

a. If f is continuous on $[0, \pi]$, show that

$$\int_0^{\pi} x f(\sin x) dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$$

b. Use a. to evaluate the integral

$$\int_0^{\pi} x \sin^2 x \cos^4 x dx$$