

Calculus Quiz 11

1. (5 pts) Let $f(x) = \int_0^x \frac{dt}{\sqrt{1+t^3}}$, $x > 0$.

a. Show that $f(x)$ has inverse function.

b. Let $g(x)$ be the inverse function of $f(x)$. Show that g has the following property

$$g''(x) = cg(x)^2$$

and determine the value c .

1. (5 pts)

a. The *error function* is defined as $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.

Show that the function $y = e^{x^2} \operatorname{erf}(x)$ satisfies the differential equation

$$y' = 2xy + \frac{2}{\sqrt{\pi}}$$

b. Evaluate the indefinite integral $\int \frac{dx}{\csc x - 2 \cot x}$.