

# Calculus Quiz 12

1. (5 pts) A population often increases exponentially in its early stages but levels off eventually and approaches its carrying capacity because of limited resources. Let  $P(t)$  is the size of the population at time  $t$ ., we assume that  $P$  satisfies following equation

$$\frac{1}{P} \frac{dP}{dt} = k \left( 1 - \frac{P}{A} \right), \text{ where } A, k \text{ are constants}$$

- a. By letting  $Q(t) = \frac{A}{P(t)} - 1$ , then what kind of equation that  $Q$  have to be satisfied?
- b. Prove that  $P(t) = \frac{A}{1 + Be^{-kt}}$  for some constant  $B$ .

2. (5 pts)

a. For  $x \geq 0$ , show that

$$\sin^{-1} \left( \frac{x-1}{x+1} \right) - 2 \tan^{-1} \sqrt{x} = C$$

for some constant  $C$  and determine what value  $C$  is.

b. Recall that  $\int_0^\pi x f(\sin x) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$  for continuous  $f$  defined on  $[0, \pi]$ . Use this to evaluate the integral

$$\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$$