

Calculus Quiz 15

1. (5 pts)

a. Use the Trapezoidal Rule with $n = 10$ to approximate

$\int_0^{20} \cos(\pi x) dx$. Compare your result to the actual value.
Can you explain the discrepancy?

b. Let f be a polynomial with $\deg f = 3$ or lower, which defined on $[a, b]$. Show that the Simpson's Rule gives the

exact value of $\int_a^b f(x) dx$ [*Hint*: it suffice to show the result when there are two subintervals ($n = 2$), since for a larger even number of subintervals the sum of exact estimates is exact.]

2. (5 pts) *The extension of factorial to non-integer values.*

By the fact that the improper integral $\int_0^{\infty} t^{x-1} e^{-t} dt$ is convergent for $x > 0$. We define it as a function of x , called the Gamma function $\Gamma(x)$.

a. Show that $\Gamma(x+1) = x\Gamma(x)$ for $x > 0$, in particular $\Gamma(n+1) = n!$ when n is positive integer.

b. Have known that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. Find $\Gamma\left(\frac{3}{2}\right)$.