Calculus Homework Assignment 5

Class: ____________________________
Student Number: ____________________
Name: _____________________________

1. Use the guidelines of this section to sketch the curve.

\[ y = x(x - 4)^3 \]

\( \text{-domain: } \mathbb{R} \)

\( f(x) = x(x - 4)^3 \Rightarrow f(0) = 0, f(4) = 0 \) are roots of order 3.

Moreover, we can see that \( f(x) < 0 \) if \( 0 < x < 4 \).

\( f(x) = 4(x-1)(x-4)^2 \Rightarrow f(1) = 0 \Rightarrow f'(x) = 12(x-1)(x-4) \).

\( f''(x) = 12(2x-1) \) 

\( f(1) \) is a local maximum.

2. Use the guidelines of this section to sketch the curve. In guideline D find an equation of the slant asymptote.

\[ y = \frac{x^2}{x - 1} \]

\( f(x) = \frac{x^2}{x - 1} = x + 1 + \frac{1}{x - 1} \)

Domain: \( \mathbb{R} \setminus \{1\} \)

\( f(0) = 0 \) is a root of order 3.

\( \lim_{x \to 1^-} f(x) = -\infty \), \( \lim_{x \to 1^+} f(x) = \infty \)

\( \lim_{x \to \infty} f(x) = \infty \), \( \lim_{x \to -\infty} f(x) = -\infty \)

\( f'(x) = \frac{2x(x-1) - x^2}{(x-1)^2} = \frac{3x^2 - 4x}{(x-1)^2} \)

\( f'(1) = 0 \) is a critical point of order 2.

\( f''(x) = \frac{6(x-1)}{(x-1)^2} = 6(x-1) \)

\( f''(1) = 0 \) is a point of inflection at \( x = 1 \).

3. A rectangular storage container with an open top is to have a volume of 10 m\(^3\). The length of its base is twice the width. Material for the base costs $10 per square meter. Material for the sides costs $6 per square meter. Find the cost of materials for the cheapest such container.

\[ 3.7 \#16 \]

\[ V = 10 \]

\[ (x, y, z) \text{ where } x = 2y \]

\[ M = 10xy + 6(0.5y^2 + 2xy) \]

\[ M(1, 2) = 20y + 24y \]

\[ M(y) = 20y + 24y \]

\[ M'(y) = 40y - \frac{180}{y^2} \]

\[ M'(y) = 0 \Rightarrow y = 3 \]

\[ M(3) = 20 \cdot 3^2 + \frac{60}{3} \]

4. Find the points on the ellipse \[ 4x^2 + y^2 = 4 \] that are farthest away from the point \((1, 0)\).

\[ 3.7 \#21 \]

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

\[ y = \frac{\pm \sqrt{4 - 4x^2}}{2} \]

\[ y = 2 - 2x \]

\[ y' = -2 \]

\[ x = \frac{-1}{2} \text{ is a point of extremum.} \]

\[ y = 0 \]

\[ (-\frac{1}{2}, 2) \]

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5. Find the most general antiderivative of the function. (Check your answer by differentiation.)

a. \( g(t) = \frac{1 + t + t^2}{\sqrt{t}} \)

\[ \Rightarrow \int g(t) \, dt = \int \left( \frac{1}{\sqrt{t}} + \frac{t}{\sqrt{t}} + \frac{t^2}{\sqrt{t}} \right) \, dt = \frac{2}{3} t^{3/2} + \frac{2}{3} t^{5/2} + \frac{2}{5} t^{5/2} + C \]

where \( C \) is a constant. \( \Box \)

b. \( h(\theta) = 2 \sin \theta - \sec^2 \theta \) \( \quad \) \([\text{§3.9 #13,15}]\)

\[ \Rightarrow \int h(\theta) \, d\theta = -2 \cos \theta - \tan \theta + C \]

where \( C \) is a constant. \( \Box \)

\( (\pi - \frac{\pi}{2} < \theta < \pi + \frac{\pi}{2}, \theta \in \mathbb{R}) \)

6. What constant acceleration is required to increase the speed of a car from 50km/h to 90km/h in 5s?

\[ 5.0 \; \text{km/h} = 50 \times \frac{1000}{3600} = \frac{5000}{36} \; \text{m/s} \]

\[ 90 \; \text{km/h} = 90 \times \frac{1000}{3600} = \frac{9000}{36} \; \text{m/s} \]

\[ \Rightarrow v = \dot{a}t + \frac{500}{36} \]

\[ \Rightarrow \frac{9000}{36} = 5 \cdot \dot{a} + \frac{500}{36} \]

\[ \Rightarrow \frac{4000}{36} = 5 \cdot \dot{a} \]

\[ \Rightarrow \dot{a} = \frac{80}{36} = \frac{20}{9} \; \text{m/s}^2 \]

7. a. Use Definition 2 to find an expression for the area under the graph of \( f \) as a limit. Do not evaluate the limit.

\[ f(x) = \frac{2x}{x^2 + 1}, \quad 1 \leq x \leq 3 \]

b. Determine a region whose area is equal to the given limit. Do not evaluate the limit.

\[ \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2}{n} \left( \frac{5 + \frac{2i}{n}}{n} \right)^{10} \]

\[ (a) \quad \Delta x = \frac{3 - 1}{n} \]

\[ \Rightarrow \int_{1}^{3} f(x) \, dx = 1 + \frac{2}{3} \]

\[ \Rightarrow A = \lim_{n \to \infty} R_n = \lim_{n \to \infty} \frac{2}{n} \sum_{i=1}^{n} \frac{2}{n} \left( 1 + \frac{2i}{n} \right)^{10} \]

\[ \text{(b)} \quad \frac{\Delta x}{2} + \frac{\Delta x}{2} \rightarrow \int_{1}^{3} (f(x))^{10} \, dx = \left( 3 - \frac{1}{2} + \frac{1}{2} \right)^{10} \]

\[ \text{Hence } f(1): (3 - 1 + 1)^{10} \text{ for } x \in [1, 3] \]

8. a. Use Definition 2 to find an expression for the area under the curve \( y = x^3 \) from 0 to 1 as a limit.

b. The following formula for the sum of the cubes of the first \( n \) integers is proved in Appendix E. Use it to evaluate the limit in part (a).

\[ 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2 \]

\[ (a) \quad \Delta x = \frac{1}{n} \Rightarrow x_i = 0 + \frac{1}{n} \]

\[ \Rightarrow A = \lim_{n \to \infty} R_n = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{1}{n} \right]^3 \frac{1}{n^2} \]

\[ = \lim_{n \to \infty} \left( \frac{1 + \frac{n}{n^2}}{2} \right) = \frac{1}{4} \]

(b) \[ A = \lim_{n \to \infty} x^3 \cdot \frac{1}{n^4} = \lim_{n \to \infty} \left( \frac{n(n+1)}{2} \right)^2 \cdot \frac{1}{n^4} \]

\[ = \lim_{n \to \infty} \left( \frac{n^2 + n}{2n^2} \right)^2 = \frac{1}{4} \]