

**112. (a)** If  $\lim_{x \rightarrow c} |f(x)| = 0$ , then  $\lim_{x \rightarrow c} [-|f(x)|] = 0$ .

$$-|f(x)| \leq f(x) \leq |f(x)|$$

$$\lim_{x \rightarrow c} [-|f(x)|] \leq \lim_{x \rightarrow c} f(x) \leq \lim_{x \rightarrow c} |f(x)|$$

$$0 \leq \lim_{x \rightarrow c} f(x) \leq 0$$

Therefore,  $\lim_{x \rightarrow c} f(x) = 0$ .

**(b)** Given  $\lim_{x \rightarrow c} f(x) = L$ :

For every  $\varepsilon > 0$ , there exists  $\delta > 0$  such that

$|f(x) - L| < \varepsilon$  whenever  $0 < |x - c| < \delta$ . Since

$||f(x)| - |L|| \leq |f(x) - L| < \varepsilon$  for

$|x - c| < \delta$ , then  $\lim_{x \rightarrow c} |f(x)| = |L|$ .