6. $\dfrac{\sqrt{a+bx}-\sqrt{3}}{x} = \dfrac{\sqrt{a+bx}-\sqrt{3}}{x} \cdot \dfrac{\sqrt{a+bx}+\sqrt{3}}{\sqrt{a+bx}+\sqrt{3}} = \dfrac{(a+bx)-3}{x\left(\sqrt{a+bx}+\sqrt{3}\right)}$

Letting $a = 3$ simplifies the numerator.

So, $\displaystyle\lim_{x\to 0}\dfrac{\sqrt{3+bx}-\sqrt{3}}{x} = \lim_{x\to 0}\dfrac{bx}{x\left(\sqrt{3+bx}+\sqrt{3}\right)} = \lim_{x\to 0}\dfrac{b}{\sqrt{3+bx}+\sqrt{3}}$

Setting $\dfrac{b}{\sqrt{3}+\sqrt{3}} = \sqrt{3}$, you obtain $b = 6$. So, $a = 3$ and $b = 6$.