2. Let (a, a^2) and $(b, -b^2 + 2b - 5)$ be the points of tangency. For $y = x^2$, $y' = 2x$ and for $y = -x^2 + 2x - 5$, $y' = -2x + 2$. So, $2a = -2b + 2 \Rightarrow a + b = 1$, or $a = 1 - b$. Furthermore, the slope of the common tangent line is

$$\frac{a^2 - (-b^2 + 2b - 5)}{a - b} = \frac{(1 - b)^2 + b^2 - 2b + 5}{(1 - b) - b} = -2b + 2$$

$$\Rightarrow \frac{1 - 2b + b^2 + b^2 - 2b + 5}{1 - 2b} = -2b + 2$$

$$\Rightarrow 2b^2 - 4b + 6 = 4b^2 - 6b + 2$$

$$\Rightarrow 2b^2 - 2b - 4 = 0$$

$$\Rightarrow b^2 - b - 2 = 0$$

$$\Rightarrow (b - 2)(b + 1) = 0$$

$$b = 2, -1$$

For $b = 2$, $a = 1 - b = -1$ and the points of tangency are $(-1, 1)$ and $(2, -5)$. The tangent line has slope -2: $y - 1 = -2(x = 1) \Rightarrow y = -2x - 1$

For $b = -1$, $a = 1 - b = 2$ and the points of tangency are $(2, 4)$ and $(-1, -8)$. The tangent line has slope 4: $y - 4 = 4(x - 2) \Rightarrow y = 4x - 4$