

68. $f(x) = ax^3 + bx^2 + cx + d, \quad a \neq 0$

$$f'(x) = 3ax^2 + 2bx + c$$

The quadratic polynomial can have zero, one, or two zeros.

$$x = \frac{-2b \pm \sqrt{4b^2 - 12ac}}{6a} = \frac{-b \pm \sqrt{b^2 - 3ac}}{3a}$$

Zero critical numbers: $b^2 < 3ac$.

Example: $(a = b = c = 1, d = 0)f(x) = x^3 + x^2 + x$ has no critical numbers.

One critical number: $b^2 = 3ac$.

Example: $(a = 1, b = c = d = 0)f(x) = x^3$ has one critical number, $x = 0$.

Two critical numbers: $b^2 > 3ac$.

Example:

$(a = c = 1, b = 2, d = 0)f(x) = x^3 + 2x^2 + x$ has two critical numbers: $x = -1, -\frac{1}{3}$.