

**94.** Let  $f$  and  $g$  be one-to-one functions.

(a) Let

$$(f \circ g)(x_1) = (f \circ g)(x_2)$$

$$f(g(x_1)) = f(g(x_2))$$

$$g(x_1) = g(x_2) \quad (\text{Because } f \text{ is one-to-one.})$$

$$x_1 = x_2 \quad (\text{Because } g \text{ is one-to-one.})$$

So,  $f \circ g$  is one-to-one.

(b) Let  $(f \circ g)(x) = y$ , then  $x = (f \circ g)^{-1}(y)$ . Also:

$$(f \circ g)(x) = y$$

$$f(g(x)) = y$$

$$g(x) = f^{-1}(y)$$

$$x = g^{-1}(f^{-1}(y))$$

$$x = (g^{-1} \circ f^{-1})(y)$$

So,  $(f \circ g)^{-1}(y) = (g^{-1} \circ f^{-1})(y)$  and

$$(f \circ g)^{-1} = g^{-1} \circ f^{-1}.$$