

$$\begin{aligned} 38. \quad \frac{1}{1-\cos \theta} &= \frac{1}{1-\cos \theta} \cdot \frac{1+\cos \theta}{1+\cos \theta} = \frac{1+\cos \theta}{1-\cos^2 \theta} \\ &= \frac{1+\cos \theta}{\sin^2 \theta} = \csc^2 \theta + \csc \theta \cot \theta \end{aligned}$$

$$\int \frac{1}{1-\cos \theta} d\theta = \int (\csc^2 \theta + \csc \theta \cot \theta) d\theta$$

$$= -\cot \theta - \csc \theta + C$$

$$= -\frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} + C$$

$$= -\frac{\cos \theta + 1}{\sin \theta} + C$$