

$$\begin{aligned}
78. \quad \frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x} &= \frac{\sin x(1 + \sin x) + \cos^2 x}{\cos x(1 + \sin x)} \\
&= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)} \\
&= \frac{\sin x + 1}{\cos x(1 + \sin x)} \\
&= \frac{1}{\cos x} = \sec x
\end{aligned}$$

So,

$$\begin{aligned}
\int \sec x \, dx &= \int \left[\frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x} \right] dx \\
&= -\ln|\cos x| + \ln|1 + \sin x| + C \\
&= \ln \left| \frac{1 + \sin x}{\cos x} \right| + C \\
&= \ln|\sec x + \tan x| + C
\end{aligned}$$