

32. Let $u = e^x$, $du = e^x dx$.

$$\frac{1}{(u^2 + 1)(u - 1)} = \frac{A}{u - 1} + \frac{Bu + C}{u^2 + 1}$$

$$1 = A(u^2 + 1) + (Bu + C)(u - 1)$$

When $u = 1$, $A = \frac{1}{2}$.

When $u = 0$, $1 = A - C$.

When $u = -1$, $1 = 2A + 2B - 2C$.

Solving these equations you have $A = \frac{1}{2}$, $B = -\frac{1}{2}$, and $C = -\frac{1}{2}$.

$$\begin{aligned} \int \frac{e^x}{(e^{2x} + 1)(e^x - 1)} dx &= \int \frac{1}{(u^2 + 1)(u - 1)} du \\ &= \frac{1}{2} \left(\int \frac{1}{u - 1} du - \int \frac{u + 1}{u^2 + 1} du \right) \\ &= \frac{1}{2} \left(\ln|u - 1| - \frac{1}{2} \ln|u^2 + 1| - \arctan u \right) + C \\ &= \frac{1}{4} \left(2 \ln|e^x - 1| - \ln|e^{2x} + 1| - 2 \arctan e^x \right) + C \end{aligned}$$