# Calc. Homework Assignment-MGT1

**Class:** ____________________

**Student Number:** ____________________

**Name:** ____________________

**1.** Find each indefinite integral.

**a.** $\int (e^x - x)^2 dx$ **b.** $\int \sqrt{x} \ln \sqrt{x} dx$

[§7.1 #5, 17]

$\int (e^{2x} - 2xe^x + x^2)\, dx$

$\int e^{2x} dx - 2\int xe^x dx + \int x^2 dx$ (Let $u = x$, $dv = e^x dx$, $du = dx$, $v = e^x$)

$\frac{1}{2}e^{2x} - 2(xe^x - \int e^x dx) + \frac{x^3}{3} + C_1$

$\frac{1}{2}e^{2x} - 2(xe^x - e^x) + \frac{x^3}{3} + C$ #

b. Let $u = \ln x^{\frac{1}{2}} = \frac{1}{2}\ln x$, $dv = x^{\frac{1}{2}} dx$.

$du = \frac{1}{2x} dx$, $v = \frac{2}{3} x^{\frac{3}{2}}$

$\int u\,dv = uv - \int v\,du = \frac{2}{3}x^{\frac{3}{2}}(\ln\sqrt{x}) - \int \frac{2}{3}x^{\frac{3}{2}} \cdot \frac{1}{2x} dx$

$= \frac{2}{3}x^{\frac{3}{2}}(\ln\sqrt{x}) - \frac{1}{3}\int x^{\frac{1}{2}} dx = \frac{2}{3}x^{\frac{3}{2}}(\ln\sqrt{x}) - \frac{2}{9}x^{\frac{3}{2}} + C$ #

**2.** The membership of the Cambridge Community Health Plan (a health maintenance organization) is projected to grow at the rate of

$$9\sqrt{t+1}\ln\sqrt{t+1}$$ 101.606

thousand people/year, $t$ years from now. If the HMO's current membership is 50000, what will be the membership 5 years from now? [§7.1 #45]

$9\int_0^5 \sqrt{t+1} \cdot \ln\sqrt{t+1}\, dt$ (Let $x = t+1$, $dx = dt$)

$= 9\int_1^6 \sqrt{x} \cdot \ln\sqrt{x}\, dx$ by 1.(b.)

$= 9\left(\frac{2}{3}x^{\frac{3}{2}}(\ln\sqrt{x}) - \frac{2}{9}x^{\frac{3}{2}}\right)\Big|_1^6$

$= 6(t+1)^{\frac{3}{2}}\ln\sqrt{t+1} - 2(t+1)^{\frac{3}{2}}\Big]_0^5$

$= 6 \cdot 6^{\frac{3}{2}} \cdot \ln 6^{\frac{1}{2}} - 2 \cdot 6^{\frac{3}{2}} + 2$

$\approx 51.606$

$50 + 51.606 = 101.606$ (thousand).

**3.** Use the table of integrals to find or evaluate each integral.

**a.** $\int_0^2 \frac{dx}{\sqrt{9+4x^2}}$ **b.** $\int \frac{3e^x}{1+e^{x/2}} dx$

[§7.2 #9, 23]

a. use Formula (9) with $a = 3$, $u = 2x$, $\Rightarrow du = 2dx$

$\int_0^2 \frac{dx}{\sqrt{9+4x^2}} = \frac{1}{2}\int_0^4 \frac{du}{\sqrt{3^2+u^2}} = \frac{1}{2}\ln|u + \sqrt{9+u^2}|\Big|_0^4$

$= \frac{1}{2}(\ln 9 - \ln 3) = \frac{1}{2}\ln 3$

b. Let $v = e^{x/2}$, $dv = \frac{1}{2}e^{x/2} dx$

$\int \frac{3e^x}{1+e^{x/2}} dx = 3\int \frac{e^{x/2}}{e^{-x/2}+1} dx = 6\int \frac{dv}{(1/v)+1}$

$= 6\int \frac{v\,dv}{1+v} = 6(1+v - \ln|1+v|) + C$

$= 6[1+e^{x/2} - \ln(1+e^{x/2})] + C$

**4.** One reason for the increase in the life span over the years has been the advances in medical technology. The average life span for American women from 1907 through 2007 is given by

$$W(t) = 49.9 + 17.1\ln t \qquad (1 \le t \le 6)$$

where $W(t)$ is measured in years and $t$ is measured in 20-year intervals, with $t = 1$ corresponding to 1907. What is the average average life expectance for women from 1907 through 2007? [§7.2 #37]

$\frac{1}{5}\int_1^6 49.9 + 17.1\ln t\, dt$

$= \frac{1}{5}\left[49.9t\Big|_1^6 + 17.1\int \ln t\, dt\right]$

$= \frac{1}{5}\left[249.5 + 17.1(t\ln t - t)\Big|_1^6\right]$

$= \frac{1}{5}[102.6\ln 6 + 164]$

$\approx 69.6$ (years)

**5.** Find a bound on the error in approximating

$$\int_1^3 \frac{1}{x}\,dx$$

with 10 intervals.

**a.** Using the Trapezoidal Rule. 0.013

**b.** Using the Simpson's Rule. 0.00043

[§7.3 #25]

a. error bound $= \dfrac{M(b-a)^3}{12n^2}$

$a=1,\ b=3,\ n=10$

$f(x)=\frac{1}{x},\ f'(x)=\frac{-1}{x^2},\ f''(x)=\frac{2}{x^3},\ f^{(3)}(x)=\frac{-6}{x^4}<0$

take $M=f''(1)=2$

$\dfrac{M(b-a)^3}{12n^2} = \dfrac{2(3-1)^3}{12(100)} \approx 0.013$

b. $f^{(4)}(x)=\dfrac{24}{x^5}$, $f^{(5)} = \dfrac{-120}{x^6}<0$

$f^{(4)}(x)$ is decreasing, take $M=f^{(4)}(1)=24$

error bound $= \dfrac{24(3-1)^5}{180(10^4)} \approx 0.00043$

**6.** According to data from the American Petroleum Institute, the U.S. strategic petroleum reserves from the beginning of 2002 through the beginning of 2012 can be approximated by the function

$$S(t)=\frac{720t^2+3480}{t^2+6.3} \qquad (0\le t\le 10)$$

where $S(t)$ is measured in millions of barrels and $t$ in years, with $t=0$ corresponding to the beginning of 2002. Using the Trapezoidal Rule with $n=10$, estimate the average petroleum reserves from the beginning of 2002 through the beginning of 2012. [§7.3 #37]

$A=\dfrac{1}{10-0}\displaystyle\int_0^{10} S(t)\,dt = \frac{1}{10}\int_0^{10}\frac{720t^2+3480}{t^2+6.3}\,dt$

Using the Trapezoidal Ruel with $a=0,\ b=10$ and $n=10$, so that $\Delta t=\frac{10-0}{10}=1$

we have $t_i=i\ \ \forall\ i=0,1,\dots,10$

Thus,

$A=\dfrac{1}{10}\displaystyle\int_0^{10} S(t)\,dt$

$=\left(\frac{1}{10}\right)\left(\frac{1}{2}\right)[S(0)+2S(1)+2S(2)+\dots+2S(9)+S(10)]$

$\approx 664.27$ (million barrels).

**7.** Evaluate each improper integral whenever it is convergent.

**a.** $\displaystyle\int_4^\infty \frac{2}{x^{3/2}}\,dx$

**b.** $\displaystyle\int_{-\infty}^0 xe^x\,dx$

[§7.4 #21, 35]

a. $\displaystyle\int_4^\infty \frac{2}{x^{3/2}}\,dx = \lim_{b\to\infty}\int_4^b \frac{2}{x^{3/2}}\,dx$

$=\displaystyle\lim_{b\to\infty}(-4x^{-\frac{1}{2}})\Big|_4^b = \lim_{b\to\infty}\left(\frac{-4}{\sqrt{b}}+2\right)=2$

b. $\displaystyle\int_{-\infty}^0 xe^x\,dx = \lim_{b\to-\infty}\int_b^0 xe^x\,dx$

$=\displaystyle\lim_{b\to-\infty}(xe^x-e^x)\Big|_b^0 = \lim_{b\to-\infty}(-1)-be^b+e^b$

$=-1$

**8.** The capital value (present sale value) $CV$ of property that can be rented on a perpetual basis for $R$ dollars annually is given by

$$CV\approx\int_0^\infty Re^{-it}\,dt$$

where $i$ is the prevailing continuous interest rate.

**a.** Show that $CV\approx R/i$.

**b.** Find the capital value of property that can be rented at \$10000 annually when the prevailing continuous interest rate is $6\%/year$.

[§7.4 #57]

a. $\displaystyle\int_0^\infty Re^{-it}\,dt = \lim_{b\to\infty}\int_0^b Re^{-it}\,dt$

$=\displaystyle\lim_{b\to\infty} -R\frac{1}{i}e^{-it}\Big|_0^b = \lim_{b\to\infty}\left(-\frac{R}{i}e^{-ib}+\frac{R}{i}\right)$

$=\dfrac{R}{i}$

b. $CV=\dfrac{R}{i}=\dfrac{10000}{0.06}\approx 166667$ (dollar