Calc. Homework Assignment-MGT2

Class: ______________________

Student Number: ______________________

Name: ______________________

1. Find the volume of the solid obtained by revolving the region under the curve $y = e^x$ from $x = 0$ to $x = 1$ about the x-axis.
[§7.5 #13]

Sol:

$$\pi\int_0^1 (e^x)^2\,dx$$

$$= \pi\int_0^1 e^{2x}\,dx$$

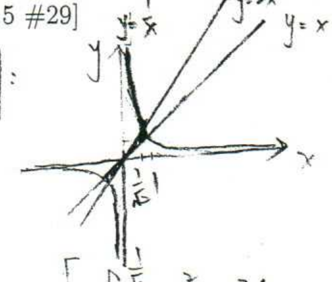
$$= \pi\cdot\frac{1}{2}e^{2x}\Big|_0^1$$

$$= \frac{(e^2-1)}{2}\pi$$

#.

3. Find the volume of the solid obtained by revolving the region bounded by the graphs of the functions $y = \frac{1}{x}$, $y = x$ and $y = 2x$ about the x-axis.
[§7.5 #29]

Sol:



$$2\times\pi\left[\int_0^{\frac{1}{\sqrt{2}}}[(2x)^2-x^2]\,dx + \int_{\frac{1}{\sqrt{2}}}^{1}[(\tfrac{1}{x})^2-x^2]\,dx\right]$$

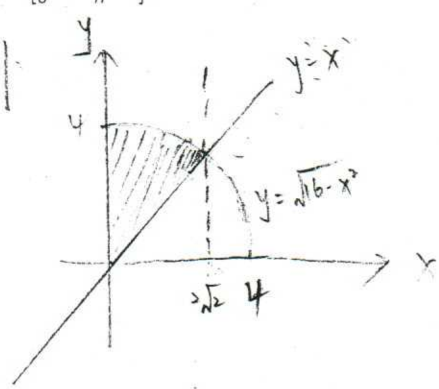
$$= 2\times\pi\left[\int_0^{\frac{1}{\sqrt{2}}}3x^2\,dx + \int_{\frac{1}{\sqrt{2}}}^{1}(\tfrac{1}{x^2}-x^2)\,dx\right]$$

$$= 2\times\pi\left[x^3\Big|_0^{\frac{1}{\sqrt{2}}} + \left[(-\tfrac{1}{x})-\tfrac{x^3}{3}\right]\Big|_{\frac{1}{\sqrt{2}}}^{1}\right] = \frac{8\sqrt{2}-8}{3}\pi$$

#.

2. Find the volume of the solid obtained by revolving the region bounded above by the curve $y = \sqrt{16-x^2}$ and below by the curve $y = x$ from $x = 0$ to $x = 2\sqrt{2}$ about the x-axis.
[§7.5 #19]

Sol:



$$\pi\int_0^{2\sqrt{2}}\left[(\sqrt{16-x^2})^2 - x^2\right]dx$$

$$= \pi\int_0^{2\sqrt{2}} 16-2x^2\,dx$$

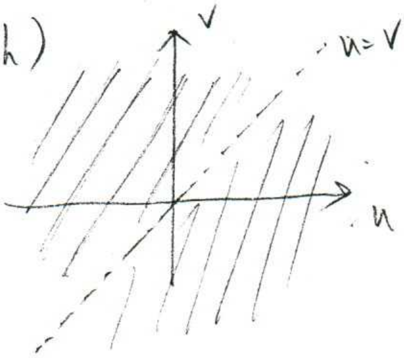
$$= \pi\left(16x-\frac{2}{3}x^3\Big|_0^{2\sqrt{2}}\right)$$

$$= \frac{64\sqrt{2}}{3}\pi$$

4. Find the domain of the function.
a. $h(u,v) = \dfrac{uv}{u-v}$
b. $h(x,y) = \ln(x+y-5)$
[§8.1 #13, 17]

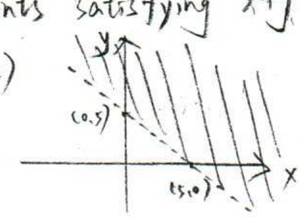
Sol: a. $h(u,v)$ is defined for all $u \neq v$.
The domain of h is the set of all points in the uv-plane except $u=v$ #

(Domain of h)



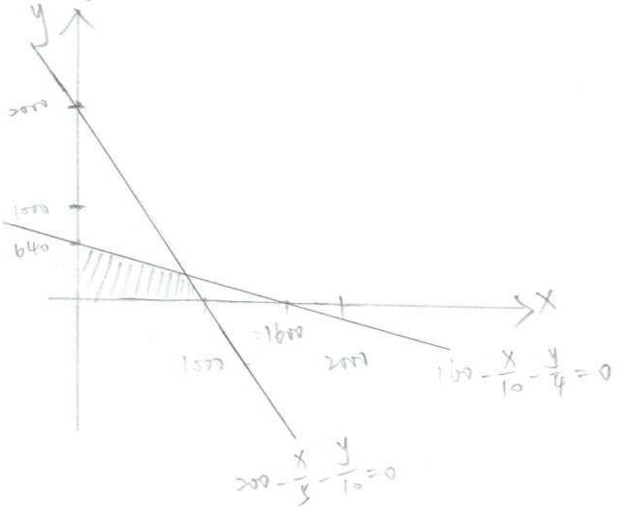
b. $\because x+y-5>0$
$\therefore$ The domain of h is the set of all points satisfying $x+y>5$

(Domain of h) #



(Over Please)

Figure 5(b)



y
x
2000
1500
640
1000
1600
2000
$160 - \frac{x}{10} - \frac{y}{4} = 0$
$200 - \frac{x}{5} - \frac{y}{10} = 0$

5. Country Workshop manufactures both finished and unfinished furniture for the home. The estimated quantities demanded each week of its rolltop desks in the finished and unfinished versions are x and y units when the corresponding unit prices are

$$p = 200 - \frac{1}{5}x - \frac{1}{10}y$$

$$q = 160 - \frac{1}{10}x - \frac{1}{4}y$$

dollars, respectively.
a. What is the weekly total revenue function $R(x,y)$?
b. Find the domain of the function R.
[§8.1 #39]

a. $R(x,y) = Px + qy = x(200 - \frac{x}{5} - \frac{y}{10}) + y(160 - \frac{x}{10} - \frac{y}{4})$

$= -\frac{1}{5}x^2 - \frac{1}{4}y^2 - \frac{1}{5}xy + 200x + 160y$

b. $\because P, q \geq 0 \therefore (200 - \frac{x}{5} - \frac{y}{10}) \geq 0, (160 - \frac{x}{10} - \frac{y}{4}) \geq 0$

Domain of the function $= \begin{cases} x \geq 0 \\ y \geq 0 \\ 200 - \frac{x}{5} - \frac{y}{10} \geq 0 \\ 160 - \frac{x}{10} - \frac{y}{4} \geq 0 \end{cases}$

6. Find the first partial derivatives of the function.
a. $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$
b. $f(x,y) = (x^2 + y^2)^{2/3}$
[§8.2 #10, 13]

sol:

a. $f_x = \frac{\partial f}{\partial x} = \frac{2x(x^2+y^2) - (x^2-y^2)\cdot 2x}{(x^2+y^2)^2} = \frac{4xy^2}{(x^2+y^2)^2}$

$f_y = \frac{\partial f}{\partial y} = \frac{-2y(x^2+y^2) - (x^2-y^2)\cdot 2y}{(x^2+y^2)^2} = \frac{-4yx^2}{(x^2+y^2)^2}$ #

b. $f_x = \frac{\partial f}{\partial x} = \frac{2}{3}(x^2+y^2)^{-\frac{1}{3}} \cdot 2x = \frac{4}{3}x \cdot (x^2+y^2)^{-\frac{1}{3}}$

$f_y = \frac{\partial f}{\partial y} = \frac{2}{3}(x^2+y^2)^{-\frac{1}{3}} \cdot 2y = \frac{4}{3}y(x^2+y^2)^{-\frac{1}{3}}$ #

7. Evaluate the first partial derivatives of the function at the given point.
a. $f(x,y) = x\sqrt{y} + y^2; (2,1)$
b. $f(x,y,z) = x^2yz^3; (1,0,2)$
[§8.2 #27, 33]

a. $f_x(2,1) = \sqrt{y}\,\Big|_{x=2, y=1} = 1$

$f_y(2,1) = \frac{x}{2\sqrt{y}} + 2y\,\Big|_{x=2, y=1} = 1 + 2 = 3$ #

b. $f_x(1,0,2) = y \cdot z^3 \cdot 2x\,\Big|_{(1,0,2)} = 0$

$f_y(1,0,2) = x^2z^3\,\Big|_{(1,0,2)} = 1 \cdot 2^3 = 8$

$f_z(1,0,2) = 3x^2yz^2\,\Big|_{(1,0,2)} = 0$ #

8. The efficiency of an internal combustion engine is given by

$$E = (1 - \frac{v}{V})^{0.4}$$

where V and v are the respective maximum and minimum volumes of air in each cylinder.
a. Show that $\partial E/\partial V > 0$,and interpret your result.
b. Show that $\partial E/\partial v < 0$,and interpret your result.
[§8.2 #55]

a. $\frac{\partial E}{\partial V} = 0.4(1 - \frac{v}{V})^{-0.6} \cdot \frac{v}{V^2}$

$\because \frac{v}{V} < 1 \therefore \frac{\partial E}{\partial V} > 0$

b. $\frac{\partial E}{\partial v} = 0.4(1 - \frac{v}{V})^{-0.6}(-\frac{1}{V}) < 0$