Calc. Homework Assignment-MGT3

Class: ____________________

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1. Find the critical point(s) of the function $f(x, y) = xy + \frac{4}{x} + \frac{2}{y}$. Then use the second derivative test to classify the nature of each point, if possible. Finally, determine the relative extrema of the function.
[§8.3 #13]

$$\begin{cases} f_x(x,y) = y - \frac{4}{x^2} = 0 \Rightarrow x^2 y = 4 \\ f_y(x,y) = x - \frac{2}{y^2} = 0 \Rightarrow x y^2 = 2 \end{cases}$$

$\Rightarrow \underline{(x,y) = (2,1)}$. #

$f_{xx}(x,y) = \frac{8}{x^3}$, $f_{xx}(2,1) = 1 > 0$

$f_{yy}(x,y) = \frac{4}{y^3}$, $f_{yy}(2,1) = 4$

$f_{xy}(x,y) = 1$, $f_{xy}(2,1) = 1$

$D(2,1) = 1 \cdot 4 - 1^2 = 3 > 0$

$\Rightarrow$ relative minimum value:

$f(2,1) = 2+2+2 = 6$. #

2. Find the critical point(s) of the function $f(x, y) = \ln(1 + x^2 + y^2)$. Then use the second derivative test to classify the nature of each point, if possible. Finally, determine the relative extrema of the function.
[§8.3 #19]

$$\begin{cases} f_x(x,y) = \frac{2x}{1+x^2+y^2} = 0 \Rightarrow x = 0 \\ f_y(x,y) = \frac{2y}{1+x^2+y^2} = 0 \Rightarrow y = 0 \end{cases}$$

$\Rightarrow \underline{(x,y) = (0,0)}$. #

$f_{xx}(x,y) = \frac{2(1+x^2+y^2) - (2x)(2x)}{(1+x^2+y^2)^2}$,

$f_{xx}(0,0) = \frac{2-0}{1} = 2 > 0$

$f_{yy}(x,y) = \frac{2(1+x^2+y^2) - (2y)(2y)}{(1+x^2+y^2)^2}$,

$f_{yy}(0,0) = \frac{2-0}{1} = 2$

$f_{xy}(x,y) = \frac{-(2x)(2y)}{(1+x^2+y^2)^2}$, $f_{xy}(0,0) = \frac{0}{1} = 0$

$D(0,0) = 2 \cdot 2 - 0^2 = 4 > 0$

$\Rightarrow$ relative minimum value:

$\underline{f(0,0) = \ln 1 = 0}$. #

3. The management of Cal Supermarkets has determined that the quantity demanded per week of their 90% lean ground sirloin, x, and the quantity demanded per week of their 80% ground beef, y(both measured in pounds), are related to their unit prices p and q (in dollar), repectively, by the equations

$$x = 6400 - 400p - 200q, \qquad y = 5600 - 200p - 400q$$

a. What is the total revenue function $R(p, q)$?
b. What price should Cal Supermarkets charge for each product to maximize its weekly revenue? How many pounds of each product will then be sold? What is the maximum revenue? [§8.3 #25]

a. $R(p,q) = xp + yq = \underline{400(-p^2 - q^2 - pq + 16p + 14q)}$.

b. $\begin{cases} R_p(p,q) = 400(-2p - q + 16) = 0 \Rightarrow 2p + q = 16 \\ R_q(p,q) = 400(-2q - p + 14) = 0 \Rightarrow p + 2q = 14 \end{cases}$

$\Rightarrow (p,q) = (6,4)$.

$R_{pp}(p,q) = -800 < 0$ $R_{qq}(p,q) = -800$ $R_{pq}(p,q) = -4$

$D(6,4) = 640000 - 160000 > 0$. $\therefore$ $6/lb, $4/lb

$6400 - 400 \cdot 6 - 200 \cdot 4 = 3200$
$5600 - 200 \cdot 6 - 400 \cdot 4 = 2800$

$6 \cdot 320$
$+ 4 \cdot 280$
$= 30400$

4. An open rectangular box having a volume of 108 in.^{3} is to be constructed from a tin sheet. Find the dimensions of such a box if the amount of material used in its construction is to be minimal.
[§8.3 #31]

Let the dimensions of the box be $x \times y \times z$.
Then $xyz = 108$, and the amount of material used is given by $S = xy + 2yz + 2xz$.

$\Rightarrow S = f(x,y) = xy + \frac{216}{x} + \frac{216}{y}$.

$$\begin{cases} f_x(x,y) = y - \frac{216}{x^2} = 0 \Rightarrow x^2 y = 216 \\ f_y(x,y) = x - \frac{216}{y^2} = 0 \Rightarrow x y^2 = 216 \end{cases}$$

$\Rightarrow (x,y) = (6,6)$

$f_{xx}(x,y) = \frac{432}{x^3}$, $f_{xx}(6,6) = 2 > 0$

$f_{yy}(x,y) = \frac{432}{y^3}$, $f_{yy}(6,6) = 2$

$f_{xy}(x,y) = 1$, $f_{xy}(6,6) = 1$

$D(6,6) = 2 \cdot 2 - 1^2 = 3 > 0$

Thus, there is a relative minimum value at $(x,y) = (6,6)$. $\Rightarrow z = 3$.

$\Rightarrow$ $\underline{6 \text{ in.} \times 6 \text{ in.} \times 3 \text{ in.}}$ #

5. End-of-year data for the number of Facebook users (in millions) from 2008 through 2011 are given in the following table:

Year,	2008	2009	2010	2011
Number, y	154.5	381.8	654.5	845.0

a. Letting $x = 0$ denote the end of 2008, find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the number of Facebook users at the end of 2015, assuming that the trend continued.
[§8.4 #11]

a. $(0^2+1^2+2^2+3^2)m+(0+1+2+3)b$
$=0\cdot154,5+1\cdot381,8+2\cdot654,5+3\cdot845,0$
$\Rightarrow 14m+6b=4225,8$ — ①
$(0+1+2+3)m+4b=154,5+381,8+654,5+845,0$
$\Rightarrow 6m+4b=2035,8$ — ②
By ① & ② $\Rightarrow m \doteq 234,4$, $b \doteq 157,3$
Thus, $y=\underline{f(x)=234,4x+157,3}$. #

b. $f(7)=234,4\cdot7+157,3=\underline{1798,1 \text{ (million)}}$. #

6. The following table gives the projected state subsidies (in millions of dollars) to the Massachusetts Bay Transit Authority (MBTA) over a 5-year period:

Year, x	1	2	3	4	5
Subsidy, y	20	24	26	28	32

a. Find an equation of the least-squares line for these data.
b. Assuming that the trend continued, estimate the state subsidy to the MBTA for the eighth year ($x = 8$).
[§8.4 #13]

a. $(1^2+2^2+3^2+4^2+5^2)m+(1+2+3+4+5)b$
$=1\cdot20+2\cdot24+3\cdot26+4\cdot28+5\cdot32$
$\Rightarrow 55m+15b=418$ — ①
$(1+2+3+4+5)m+5b=20+24+26+28+32$
$\Rightarrow 15m+5b=130$ — ②
By ① & ② $\Rightarrow m=2,8$, $b=17,6$
Thus, $y=\underline{f(x)=2,8x+17,6}$. #

b. $f(8)=2,8\cdot8+17,6=\underline{40 \text{ (million \$)}}$. #

7. Credit union membership is on the rise. The following table gives the number (in millions) of credit union members from 2003 through 2011 in 2-year intervals:

Year,	2003	2005	2007	2009	2011
Number, y	82.0	84.7	86.8	89.7	91.8

a. Letting $x = 0$ denote 2003, find an equation of the least-squares line for these data.
b. Assuming that the trend continued, estimate the number of credit union members in 2013 ($x = 5$).
[§8.4 #17]

a. $(0^2+1^2+2^2+3^2+4^2)m+(0+1+2+3+4)b$
$=0\cdot82,0+1\cdot84,7+2\cdot86,8+3\cdot89,7+4\cdot91,8$
$\Rightarrow 30m+10b=894,6$ — ①
$(0+1+2+3+4)m+5b=82,0+84,7+86,8+89,7+91,8$
$\Rightarrow 10m+5b=435$ — ②
By ① & ② $\Rightarrow m \doteq 2,5$, $b \doteq 82,1$
Thus, $y=\underline{f(x)=2,5x+82,1}$. #

b. $f(5)=2,5\cdot5+82,1=\underline{94,6 \text{ (million)}}$. #

8. The following table gives the projected spending on home care and durable medical equipment (in billions of dollars) from 2004 through 2016 ($x = 0$ corresponds to 2004):

Year, x	0	2	4	6	8	10	12
Spending, y	60	74	90	106	118	128	150

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to approximate the spending on home care and durable medical equipmant in 2015.
c. Use the result of part (a) to estimate the rate of change of the spending on home care and durable medical equipment for the period from 2004 through 2016.
[§8.4 #25]

a. $(0^2+2^2+4^2+6^2+8^2+10^2+12^2)m+(0+2+4+6+8+10+12)b=0\cdot60+2\cdot74+4\cdot90+6\cdot106+8\cdot118+10\cdot128+12\cdot150$
$\Rightarrow 364m+42b=5168$ — ①
$(0+2+4+6+8+10+12)m+7b=60+74+90+106+118+128+150$
$\Rightarrow 42m+7b=726$ — ②
By ① & ② $\Rightarrow m=7,25$, $b \doteq 60,21$
Thus, $y=\underline{f(x)=7,25x+60,21}$. #

b. $f(11)=7,25\cdot11+60,21=\underline{139,96 \text{ (billion \$)}}$. #

c. $f'(x)=\underline{7,25 \text{ (billion \$ / year)}}$. #