Calc. Homework Assignment-MGT4

Class: ______________________

Student Number: ______________________

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Yeh check

1. Find the total differential of the function.
a. $f(x, y) = \sqrt{x^2 + y^2}$ **b.** $f(x, y) = \dfrac{5y}{x - y}$
[§8.6 #5, 7]

Let $z = f(x, y)$. the total differential is dz.

a. $dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$
$= \left(\frac{1}{2}\cdot 2x(x^2+y^2)^{-\frac{1}{2}}\right)dx + \left(\frac{1}{2}\cdot 2y(x^2+y^2)^{-\frac{1}{2}}\right)dy$
$= \frac{x}{\sqrt{x^2+y^2}}dx + \frac{y}{\sqrt{x^2+y^2}}dy$ #

b. $dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$
$= \left(-5y(x-y)^{-2}\right)dx + \left(\frac{5(x-y)-5y\cdot(-1)}{(x-y)^2}\right)dy$
$= \frac{-5y}{(x-y)^2}dx + \frac{5x}{(x-y)^2}dy$ #

3. The productivity of a certain country is given by the function

$$f(x, y) = 30x^{4/5}y^{1/5}$$

when x units of labor and y units of capital are utilized. What is the approximate change in the number of units produced if the amount expended on labor is decreased from 243 to 240 units and the amount expended on capital is increased from 32 to 35 units?
[§8.6 #35]

Let $z = f(x, y)$. $dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$
$= \left(\frac{4}{5}\cdot 30x^{-\frac{1}{5}}y^{\frac{1}{5}}\right)dx + \left(\frac{1}{5}\cdot 30x^{\frac{4}{5}}y^{-\frac{4}{5}}\right)dy$
$= \left(24x^{-\frac{1}{5}}y^{\frac{1}{5}}\right)dx + \left(6x^{\frac{4}{5}}y^{-\frac{4}{5}}\right)dy$

Here $x = 243$. $dx = 240 - 243 = -3$
$y = 32$, $dy = 35 - 32 = 3$

The approximate change in z is Δz
$\Delta z \approx dz = \left(24\cdot 243^{-\frac{1}{5}}\cdot 32^{\frac{1}{5}}\right)\cdot(-3) + \left(6\cdot 243^{\frac{4}{5}}\cdot 32^{-\frac{4}{5}}\right)\cdot 3$
$= 43.125$ #

2. Find the approximate change in z when the point (x, y) changes from (x_0, y_0) to (x_1, y_1).
a. $f(x, y) = xe^{xy} - y^2$; from $(-1, 0)$ to $(-0.97, 0.03)$
b. $f(x, y) = x\ln x + y\ln x$; from $(2, 3)$ to $(1.98, 2.89)$
[§8.6 #27, 29]

Let $z = f(x, y)$

a. $dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$
$= \left((1+xy)e^{xy}\right)dx + \left(x^2e^{xy} - 2y\right)dy$

Here $x = -1$. $y = 0$.
$dx = -0.97 - (-1) = 0.03$
$dy = 0.03 - 0 = 0.03$

the approximate change in z is Δz
$\Delta z \approx dz = \left(1 + (-1\cdot 0)e^{-1\cdot 0}\right)\cdot 0.03 + \left((-1)^2e^{-1\cdot 0} - 2\cdot 0\right)\cdot 0.03$
$= 0.03 + 0.03 = 0.06$ #

b. $dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$
$= \left(\ln x + 1 + \frac{y}{x}\right)dx + (\ln x)dy$

Here $x = 2$, $y = 3$
$dx = 1.98 - 2 = -0.02$
$dy = 2.89 - 3 = -0.11$

the approximate change in z is Δz
$\Delta z \approx dz = \left(\ln 2 + 1 + \frac{3}{2}\right)\cdot(-0.02) + (\ln 2)\cdot(-0.11)$
$= -0.13\cdot\ln 2 - 0.05 \approx -0.1401$ #

4. The total resistance R of three resistors with resistance R_1, R_2, and R_3, connected in parallel, is given by the relationship

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If R_1, R_2, and R_3 are measured at 100, 200, and 300 ohms, respectively, with a maximum error of 1% in each measurement, find the approximate maximum error in the calculated value of R.
[§8.6 #47]

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \Rightarrow R = (R_1^{-1} + R_2^{-1} + R_3^{-1})^{-1}$

$dR = \frac{\partial R}{\partial R_1}dR_1 + \frac{\partial R}{\partial R_2}dR_2 + \frac{\partial R}{\partial R_3}dR_3$
$= -(R_1^{-1}+R_2^{-1}+R_3^{-1})^{-2}\cdot(-R_1^{-2})\,dR_1 +$
$-(R_1^{-1}+R_2^{-1}+R_3^{-1})^{-2}\cdot(-R_2^{-2})\,dR_2 +$
$-(R_1^{-1}+R_2^{-1}+R_3^{-1})^{-2}\cdot(-R_3^{-2})\,dR_3$
$= R^2(R_1^{-2}dR_1 + R_2^{-2}dR_2 + R_3^{-2}dR_3)$

Since the error committed in measuring R_1, R_2, R_3 is at most 1%, we have
$|\Delta R_1| \le 0.01\times 100 = 1$.
$|\Delta R_2| \le 0.01\times 200 = 2$.
$|\Delta R_3| \le 0.01\times 300 = 3$.

The approximate maximum error is
$|\Delta R| \approx |dR| \le (100^{-1}+200^{-1}+300^{-1})^{-2}(100^{-2}\cdot 1 + 200^{-2}\cdot 2 + 300^{-2}\cdot 3)$
$\doteqdot 0.5455$ ohms #

(Over Please)

5. Evaluate the double integral

$$\int_R\int f(x,y)dA$$

for the function $f(x,y) = 4xe^{2x^2+y}$ and the region R is the rectangle defined by $0 \le x \le 1$ and $-2 \le y \le 0$.
[§8.7 #7]

$$\int_R\int f(x,y)dA = \int_{-2}^{0}\left[\int_0^1 (4xe^{2x^2+y})\,dx\right]dy$$
$$= \int_{-2}^{0}\left(e^{2x^2+y}\Big|_{x=0}^{x=1}\right)dy$$
$$= \int_{-2}^{0}(e^{2+y} - e^{y})\,dy$$
$$= e^{2+y} - e^{y}\Big|_{-2}^{0}$$
$$= (e^2-1)-(1-e^{-2})$$
$$= e^2+e^{-2}-2 \ \#$$

6. Evaluate the double integral

$$\int_R\int f(x,y)dA$$

for the function $f(x,y) = \ln y$ and the region R is the rectangle defined by $0 \le x \le 1$ and $1 \le y \le e$.
[§8.7 #9]

$$\int_R\int f(x,y)\,dA = \int_1^e\left[\int_0^1 (\ln y)\,dx\right]dy$$
$$= \int_1^e \left(x \ln y\Big|_0^1\right)dy$$
$$= \int_1^e (\ln y)\,dy$$

Using integration by parts :

Let $u = \ln y$ and $dv = dy$

then $du = \frac{1}{y}dy$, $v = y$

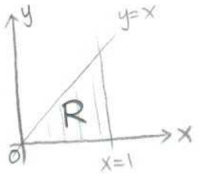
$$\int_1^e \ln y\,dy = y\ln y\Big|_1^e - \int_1^e \frac{1}{y}\cdot y\,dy$$
$$= e \quad - \quad y\Big|_1^e$$
$$= 1 \ \#$$

7. Evaluate the double integral

$$\int_R\int f(x,y)dA$$

for the function $f(x,y) = 2xe^y$ and the region R is bounded by the lines $x = 1$, $y = 0$, and $y = x$.
[§8.7 #19]

$$\int_R\int f(x,y)\,dA = \int_0^1\left[\int_0^x (2xe^y)\,dy\right]dx$$



$$= \int_0^1 \left(2xe^y\Big|_{y=0}^{y=x}\right)dx$$
$$= \int_0^1 (2xe^x - 2x)\,dx$$
$$= \int_0^1 2x(e^x-1)\,dx$$

Using integration by parts :

Let $u = 2x$. and $dv = (e^x-1)dx$,

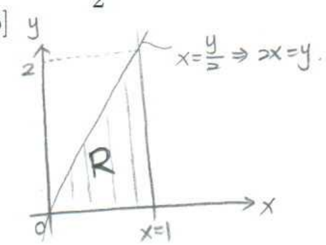
then $du = 2dx$ and $v = e^x - x$

$$\int_0^1 2x(e^x-1)\,dx = 2x(e^x-x)\Big|_0^1 - \int_0^1 2(e^x-x)\,dx$$
$$= 2(e-1) - 2\left(e^x - \tfrac{1}{2}x^2\Big|_0^1\right)$$
$$= 2(e-1) - 2\left(e-\tfrac{1}{2}-1\right)$$
$$= 1 \ \#$$

8. Evaluate the double integral

$$\int_R\int f(x,y)dA$$

for the function $f(x,y) = ye^{x^3}$ and the region R is bounded by $x = \dfrac{y}{2}$, $x = 1$, and $y = 0$.
[§8.7 #25]



$$\int_R\int f(x,y)\,dA = \int_0^1\left[\int_0^{2x}(ye^{x^3})\,dy\right]dx$$
$$= \int_0^1\left(\tfrac{1}{2}y^2e^{x^3}\Big|_0^{2x}\right)dx$$
$$= \int_0^1 (2x^2e^{x^3})\,dx$$
$$= \tfrac{2}{3}e^{x^3}\Big|_0^1$$
$$= \tfrac{2}{3}(e-1) \ \#$$