Calc. Homework Assignment-MGT5

Class: ______________________

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Yeh check

1. Find the volume of the solid bounded above by the surface $z = f(x, y) = 4 - 2x - y$ and below by the plane region $R = \{(x, y)|0 \le x \le 1; 0 \le y \le 2\}$.
[§8.8 #9]

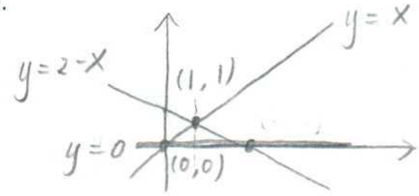
$$\iint_R f(x,y)\,dA$$
$$= \int_0^1 \left(\int_0^2 (4-2x-y)\,dy \right) dx$$
$$= \int_0^1 \left(\left(4y - 2xy - \frac{y^2}{2}\right)\Big|_0^2 \right) dx$$
$$= \int_0^1 (8-4x-2)\,dx$$
$$= \int_0^1 (6-4x)\,dx$$
$$= (6x - 2x^2)\Big|_0^1$$
$$= 4 \quad ※$$

2. Find the volume of the solid bounded above by the surface $z = f(x, y) = x^2 + y^2$ and below by the plane region R is the rectangle with vertices $(0, 0)$, $(1, 0)$, $(1, 2)$, and $(0, 2)$.
[§8.8 #11]

$$\iint_R f(x,y)\,dA$$
$$= \int_0^1 \left(\int_0^2 (x^2+y^2)\,dy \right) dx$$
$$= \int_0^1 \left(\left(x^2 y + \frac{y^3}{3}\right)\Big|_0^2 \right) dx$$
$$= \int_0^1 \left(2x^2 + \frac{8}{3}\right) dx$$
$$= \left(\frac{2x^3}{3} + \frac{8}{3}x\right)\Big|_0^1$$
$$= \frac{2}{3} + \frac{8}{3}$$
$$= \frac{10}{3} \quad ※$$

3. Find the average value of the function $f(x, y) = xy$ over the plane region R is the triangle bounded by $y = x$, $y = 2 - x$, and $y = 0$.
[§8.8 #19]

The area of $R = 1$



$$\frac{1}{1}\int_0^1 \int_y^{2-y} xy\,dx\,dy$$
$$= \int_0^1 \left(\frac{y}{2} x^2 \Big|_y^{2-y} \right) dy$$
$$= \int_0^1 \frac{y}{2}\left[(2-y)^2 - y^2\right] dy$$
$$= \int_0^1 \frac{y}{2}(4-4y)\,dy$$
$$= \int_0^1 (2y - 2y^2)\,dy$$
$$= y^2 - \frac{2}{3}y^3 \Big|_0^1 = 1 - \frac{2}{3} = \frac{1}{3} \quad ※$$

4. The population density (number of people per square mile) of a certain city is given by the function

$$f(x, y) = \frac{50,000|xy|}{(x^2 + 20)(y^2 + 36)}$$

where the origin $(0, 0)$ gives the location of the government center. Find the population inside the rectangular area described by

$$R = \{(x, y)| -15 \le x \le 15; -20 \le y \le 20\}$$

[§8.8 #25]

$$\iint_R f(x,y)\,dA$$
$$= \int_{-15}^{15} \left(\int_{-20}^{20} \frac{50000|xy|}{(x^2+20)(y^2+36)}\,dy \right) dx$$
$$= \int_{-15}^{15} \left(2\int_0^{20} \frac{50000\,xy}{(x^2+20)(y^2+36)}\,dy \right) dx$$
$$= \int_{-15}^{15} \left(\frac{50000}{x^2+20} \left(\ln|y^2+36|\right)\Big|_0^{20} \right) dx$$
$$= \int_{-15}^{15} \left(\frac{50000}{x^2+20} \cdot \ln\frac{436}{36} \right) dx$$
$$= 2\int_0^{15} \left(\frac{50000}{x^2+20} \cdot \ln\frac{109}{9} \right) dx$$
$$= 50000 \ln\frac{109}{9} \cdot \left(\ln|x^2+20|\right)\Big|_0^{15}$$

(Over Please) $= 50000 \cdot \ln\frac{109}{9} \cdot \ln\frac{245}{20} = 50000 \cdot \ln\frac{109}{9} \cdot \ln\frac{4}{}$

5. Verify that $y = e^{-2x}$ is a solution of the differential equation $y'' + y' - 2y = 0$.
[§9.1 #5]

Let $y = e^{-2x}$

Then $y' = -2e^{-2x}$

$y'' = 4e^{-2x}$

$y'' + y' - 2y$

$= 4e^{-2x} - 2e^{-2x} - 2e^{-2x}$

$= 0$ ※

6. Verify that y is a solution of the differential equation. $y = C - Ae^{-kt}$, A and C constants; $\frac{dy}{dt} = k(C - y)$
[§9.1 #11]

Let $y = C - Ae^{-kt}$

$\frac{dy}{dt} = Ake^{-kt}$

Since $k(C-y)$

$= k(C - C + Ae^{-kt})$

$= Ake^{-kt}$,

we see $\frac{dy}{dt} = k(C-y)$.

7. Verify that $y = \frac{C}{x}$ is a ~~general~~ solution of the differential equation $y' + \left(\frac{1}{x}\right)y = 0$. Then find a particular solution of the differential equation that satisfies the side condition $y(1) = 1$.
[§9.1 #15]

$y' = -\frac{C}{x^2}$

$y' + (\frac{1}{x})y$

$= -\frac{C}{x^2} + \frac{1}{x} \cdot \frac{C}{x}$

$= -\frac{C}{x^2} + \frac{C}{x^2}$

$= 0$ ※

$y = f(x) = \frac{C}{x}$

$f(1) = \frac{C}{1} = 1 \Rightarrow C = 1$

particular solution : $y = \frac{1}{x}$ ※

8. Verify that $y = \frac{Ce^x}{x} + \frac{1}{2}xe^x$ is a ~~general~~ solution of the differential equation $y' + \left(\frac{1-x}{x}\right)y = e^x$. Then find a particular solution of the differential equation that satisfies the side condition $y(1) = -\frac{1}{2}e$.
[§9.1 #17]

$y' = \frac{ce^x \cdot x - ce^x}{x^2} + \frac{1}{2}e^x + \frac{1}{2}xe^x$

$y' + (\frac{1-x}{x})y$

$= \frac{ce^x \cdot x - ce^x}{x^2} + \frac{1}{2}e^x + \frac{1}{2}xe^x + \frac{ce^x - ce^x \cdot x}{x^2} + \frac{1}{2}e^x - \frac{1}{2}xe^x$

$= e^x$ ※

$y = f(x) = \frac{ce^x}{x} + \frac{1}{2}xe^x$

$y(1) = ce + \frac{1}{2}e = -\frac{1}{2}e \Rightarrow c = -1$

particular solution : $y = -\frac{e^x}{x} + \frac{1}{2}xe^x$ ※