Calc. Homework Assignment-MGT7

Class: ____________________

Student Number: ____________________

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1. Find the nth Taylor polynomial of the function at the indicated number.

a. $f(x) = e^x$ at $x = 1, n = 4$

b. $f(x) = \dfrac{1}{2x+3}$ at $x = 0, n = 3$

[§11.1 #15, 19]

2. The registrar of Kellogg University estimates that the total student enrollment in the Continuing Education division will be given by

$$N(t) = -\frac{20,000}{\sqrt{1+0.2t}} + 21,000$$

where $N(t)$ denotes the number of students enrolled in the division t years from now. Use the second Taylor polynomial of N at $t = 0$ to approximate the average enrollment at Kellogg University between $t = 0$ and $t = 2$.
[§11.1 #41]

3. Find the nth term of the sequence. (Assume that the "obvious" pattern continues.)

a. 1, 4, 7, 10, $\cdots$

b. 2, $\dfrac{8}{5}, \dfrac{32}{25}, \dfrac{128}{125}, \cdots$

[§11.2 #11, 15]

4. Determine the convergence or divergence of the sequence $\{a_n\}$. If the sequence converges, find its limit.

a. $a_n = \dfrac{(-1)^n}{\sqrt{n}}$

b. $a_n = \dfrac{2n^4 - 1}{n^3 + 2n + 1}$

[§11.2 #33, 37]

5. Of the microprocessors manufactured by a microelectronics firm for use in regulating fuel consumption in automobiles, $1\frac{1}{2}\%$ are defective. It can be shown that the probability of getting at least one defective microprocessor in a random sample of n microprocessors is $f(n) = 1 - (0.985)^n$. Consider the sequence $\{a_n\}$ defined by $a_n = f(n)$.
a. Write down the terms a_1, a_{10}, a_{100}, and a_{1000} of the sequence $\{a_n\}$.
b. Evaluate $\lim_{n\to\infty} a_n$, and interpret your results.
[§11.2 #45]

6. Determine whether the geometric series converges or diverges. If it converges, find its sum.
a. $\sum_{n=0}^{\infty} 2(1.01)^n$
b. $\sum_{n=0}^{\infty} \frac{(-2)^n}{3^n}$
[§11.3 #7, 9]

7. Determine whether the series converges or diverges. If it converges, find its sum.
a. $\sum_{n=0}^{\infty} \frac{3 \cdot 2^n + 4^n}{3^n}$
b. $\sum_{n=1}^{\infty} \left[\left(\frac{e}{\pi}\right)^n + \left(\frac{\pi}{e^2}\right)^n\right]$
[§11.3 #23, 25]

8. Suppose that the average wage earner saves 9% of her take-home pay and spends the other 91%. Estimate the impact that a proposed \$30 billion tax cut will have on the economy over the long run due to the additional spending generated.
[§11.3 #35]

1. The nth Taylor Polynomial

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

a. $f(x)=e^x$, $f'(x)=e^x$, $f''(x)=e^x$, $f'''(x)=e^x$, $f^{(4)}(x)=e^x$.

$f(1)=e$, $f'(1)=e$, $f''(1)=e$, $f'''(1)=e$, $f^{(4)}(1)=e$.

so the 4th Taylor polynomial of $f(x)$ will be

$$P_4(x) = e + e(x-1) + \frac{e}{2!}(x-1)^2 + \frac{e}{3!}(x-1)^3 + \frac{e}{4!}(x-1)^4$$

$$= e + e(x-1) + \frac{e}{2}(x-1)^2 + \frac{e}{6}(x-1)^3 + \frac{e}{24}(x-1)^4 \quad \#$$

b. $f(x) = (2x+3)^{-1}$, $f(0) = \frac{1}{3}$

$f'(x) = (-2)(2x+3)^{-2}$, $f'(0) = -\frac{2}{9}$

$f''(x) = 8(2x+3)^{-3}$, $f''(0) = \frac{8}{27}$

$f'''(x) = (-48)(2x+3)^{-4}$, $f'''(0) = -\frac{48}{81}$

so the 3th Taylor polynomial of $f(x)$ will be

$$P_3(x) = \frac{1}{3} + \left(-\frac{2}{9}\right)x + \frac{8}{27}\cdot\left(\frac{1}{2!}\right)x^2 + \left(-\frac{48}{81}\right)\cdot\left(\frac{1}{3!}\right)x^3$$

$$= \frac{1}{3} - \frac{2}{9}x + \frac{4}{27}x^2 - \frac{8}{81}x^3 \quad \#$$

2. $N(t) = (-20000)(1+0.2t)^{-\frac{1}{2}} + 21000$, $N(0) = 1000$

$N'(t) = 2000\cdot(1+0.2t)^{-\frac{3}{2}}$, $N'(0) = 2000$

$N''(t) = (-600)(1+0.2t)^{-\frac{5}{2}}$, $N''(0) = -600$

so the 2th Taylor polynomial of $N(t)$ will be

$$P_2(t) = 1000 + 2000t + \left(\frac{-600}{2!}\right)t^2$$

$$= 1000 + 2000t - 300t^2$$

[461] average value of f over $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$

$$\frac{1}{2-0}\int_0^2 1000 + 2000t - 300t^2\,dt$$

$$= \frac{1}{2}\left(1000t + 1000t^2 - 100t^3\right)\Big|_0^2$$

$$= \frac{1}{2}(2000 + 4000 - 800) = 2600 \quad \#$$

3.

	a_1	a_2	a_3	a_4	
a.	1	4	7	10	...

$$1 \underbrace{\quad}_{3} 4 \underbrace{\quad}_{3} 7 \underbrace{\quad}_{3} 10$$

It is an arithmetic progression so a_n will be

$$a_n = 3\cdot(n-1)+1 \quad \#$$

	a_1	a_2	a_3	a_4	
b.	2	$\frac{8}{5}$	$\frac{32}{25}$	$\frac{128}{125}$	...
	$\frac{2^1}{5^0}$	$\frac{2^3}{5^1}$	$\frac{2^5}{5^2}$	$\frac{2^7}{5^3}$	

It is a geometric progression so an will be

$$a_n = \frac{2^{(2n-1)}}{5^{(n-1)}} \quad \#$$

4.

a. $\lim\limits_{n\to\infty} \frac{(-1)^n}{\sqrt{n}}$

$$\left|\lim_{n\to\infty} \frac{(-1)^n}{\sqrt{n}}\right| = \lim_{n\to\infty} \left|\frac{(-1)^n}{\sqrt{n}}\right| = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$$

Absolute value must be nonnegative number so when the value be zero the original value must be zero.

The sequence $\{a_n\}$ converges # and its limit is 0 #

6. $\lim_{n\to\infty} \dfrac{2n^4-1}{n^3+2n+1}$

$= \lim_{n\to\infty} \dfrac{2n-\frac{1}{n^3}}{1+\frac{2}{n^2}+\frac{1}{n^3}}$

$= \infty$

The sequence $\{a_n\}$ diverges #

5.

(a) $a_1 = 1 - 0.985 = 0.015$ #

$a_{10} = 1-(0.985)^{10} \approx 0.14027$ #

$a_{100} = 1-(0.985)^{100} \approx 0.77939$ #

$a_{1000} = 1-(0.985)^{1000} \approx 0.999999727$ #

(b) $\lim_{n\to\infty} 1-(0.985)^n$

$= \lim_{n\to\infty} 1 - \lim_{n\to\infty}\left(\frac{985}{1000}\right)^n$

$= 1 - 0$

$= 1$ #

When the n (sample) is very large, then the probability of getting at least one defective microprocessor in a random sample is 1. (must happen)

6, $\boxed{p748}$ Thm 2: If $|r|<1$, then $\sum_{n=0}^{\infty} ar^n$ converges (sum $=\frac{a}{1-r}$)

If $|r|>1$, then $\sum ar^n$ diverges

ⓐ $\sum_{n=0}^{\infty} 2\cdot(1.01)^n$

$|1.01| \geq 1$ so $\sum_{n=0}^{\infty} 2\cdot(1.01)^n$ diverges #

ⓑ $\sum_{n=0}^{\infty} 1\cdot\left(-\frac{2}{3}\right)^n$

$\left|-\frac{2}{3}\right|<1$ so $\sum_{n=0}^{\infty} \frac{(-2)^n}{3^n}$ converges #, and sum is

$\frac{1}{1-\left(-\frac{2}{3}\right)} = \frac{3}{5}$ #

7. ⓐ $\sum_{n=0}^{\infty} 3\cdot\left(\frac{2}{3}\right)^n+\left(\frac{4}{3}\right)^n = \sum_{n=0}^{\infty} 3\cdot\left(\frac{2}{3}\right)^n + \sum_{n=0}^{\infty} 1\cdot\left(\frac{4}{3}\right)^n$

$\left|\frac{4}{3}\right| \geq 1$ so $\sum_{n=0}^{\infty} 1\cdot\left(\frac{4}{3}\right)^n$ diverges and we can get

$\sum_{n=0}^{\infty} \frac{3\cdot 2^n + 4^n}{3^n}$ diverges #

ⓑ $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n + \sum_{n=1}^{\infty} \left(\frac{\pi}{e^2}\right)^n$

$\left|\frac{e}{\pi}\right|<1$ and $\left|\frac{\pi}{e^2}\right|<1$ so $\sum_{n=1}^{\infty}\left[\left(\frac{e}{\pi}\right)^n+\left(\frac{\pi}{e^2}\right)^n\right]$ converges #

Sum is $\frac{1}{1-\frac{e}{\pi}} + \frac{1}{1-\frac{\pi}{e^2}} - \left(\frac{e}{\pi}\right)^0 - \left(\frac{\pi}{e^2}\right)^0$

$= \frac{\pi}{\pi-e} + \frac{e^2}{e^2-\pi} - 2$

$= \frac{e^3+\pi^2-2e\pi}{(\pi-e)(e^2-\pi)}$ #

8. The 30 billion received by the original beneficiaries of the proposed tax cut, 0.91×30 billion dollars will be spent. Of the 0.91×30 billion dollars reinjected into the economy. 91% of it, or $0.91 \times 0.91 \times 30$ billion dollars, will find its way into the economy again. The process will go on ad infinitum, so this one-time proposed tax cut will result in additional spending over the years in the amount of

$$\sum_{n=1}^{\infty} (0.91)^n \cdot 30 = \frac{30 \times \frac{91}{100}}{1 - \frac{91}{100}}$$

$$= \frac{30 \times 91}{9}$$

$$= \frac{910}{3} \text{ billion} \quad \#$$