Calc. Homework Assignment-MGT8

須

Class: ______________________

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1. Use the integral test to determine whether the series

$$\sum_{n=1}^{\infty} ne^{-n}$$

is convergent or divergent.
[§11.4 #15]

Let $f(x) = x \cdot e^{-x}$

$f'(x) = e^{-x} - xe^{-x} < 0 \quad \forall x \geq 1$

$f(x)$ is a cont. positive. decreasing fn on $[1, \infty)$ and

$\int_1^{\infty} f(x)\,dx = \lim_{b \to \infty} -xe^{-x} - e^{-x} \Big|_1^b = 0 - (-e^{-1} - e^{-1})$

$= 2e^{-1}$ conv.

Use the integral test. $\sum_{n=1}^{\infty} n \cdot e^{-n}$ is conv.

2. Determine whether the p-series

$$\sum_{n=1}^{\infty} n^{-\pi}$$

is convergent or divergent.
[§11.4 #25]

$\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$ conv.

$\because p = \pi > 1$

3. Use the comparison test to determine whether the series

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2 - 1}}$$

is convergent or divergent.
[§11.4 #31]

$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}} > \sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2}} = \sum_{n=2}^{\infty} \frac{1}{n}$

Use p-Series

$\sum_{n=2}^{\infty} \frac{1}{n}$ div.

Use the comparison test

$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$ div.

4. Determine whether the series

$$\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$$

is convergent or divergent.
[§11.4 #43]

$\frac{\ln n}{\sqrt{n}} > \frac{1}{\sqrt{n}} \quad \forall n > 3$

& $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=2}^{\infty} \frac{1}{n^{\frac{1}{2}}}$ div. (p-series)

by the comparison test

$\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$ is div.

5. Find all values of p for which the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

is convergent.
[§11.4 #49]

Let $f(x)=\frac{1}{x(\ln x)^p}>0 \quad \forall x\in[2,\infty)$

$f'(x)<0 \quad \forall x\in(2,\infty)$

∴ $f(x)$ is conti. positive & decreasing function

(i) $p\neq 1$

$$\int_2^\infty f(x)dx=\lim_{b\to\infty}\frac{(\ln x)^{-p}}{x}dx$$

$$=\lim_{b\to\infty}\left[\frac{(\ln x)^{1-p}}{1-p}\right]_2^b$$

$$=\lim_{b\to\infty}\frac{(\ln b)^{1-p}-(\ln 2)^{1-p}}{1-p}$$

$$=\begin{cases}\frac{(\ln 2)^{1-p}}{p-1} & \text{if } p>1\\ \infty & \text{if } p<1\end{cases}$$

(ii) $p=1$

$$\int_2^\infty \frac{1}{x\ln x}dx=\int_{\ln 2}^\infty \frac{1}{u}du=\lim_{b\to\infty}\ln|u|\Big|_{\ln 2}^b=\infty$$

By the integral test. (i & ii)

$\sum_{n=2}^\infty \frac{1}{n(\ln n)^p}$ converges for $p>1$

6. Find the value(s) of a for which the series

$$\sum_{n=1}^{\infty}\left(\frac{a}{n+1}-\frac{1}{n+2}\right)$$

converges. Justify your answer.
[§11.4 #51]

$$S_N=\sum_{n=1}^{N}\left(\frac{a}{n+1}-\frac{1}{n+2}\right)$$

$$=\left(\frac{a}{2}-\frac{1}{3}\right)+\left(\frac{a}{3}-\frac{1}{4}\right)+\dots+\left(\frac{a}{N+1}-\frac{1}{N+2}\right)$$

$$=\frac{a}{2}+\frac{a-1}{3}+\frac{a-1}{4}+\dots+\frac{a-1}{N+1}-\frac{1}{N+2}$$

If $a=1$, then $S_N=\frac{1}{2}-\frac{1}{N+2}$ is the Nth partial sum of telescoping series that converges to $\frac{1}{2}$

If $a\neq 1$, then S_N is the Nth partial sum of series akin to the harmonic series $\sum_{n=1}^\infty \frac{1}{n}$, and in this case, the series diverges.

Therefore, the series converges only for $a=1$

7. Find the radius of convergence and the interval of convergence of the power series.

a. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n n!(x+2)^n}{2^n}$ **b.** $\displaystyle\sum_{n=2}^{\infty}\frac{(x+3)^n}{(n+1)^2}$

[§11.5 #9, 11]

a. $$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\left|\frac{\frac{n!}{2^n}}{\frac{(n+1)!}{2^{n+1}}}\right|$$

$$=\lim_{n\to\infty}\frac{2}{n+1}=0$$

the interval of convergence is $\{-2\}$.

b. $$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\left|\frac{\frac{1}{(n+1)^2}}{\frac{1}{(n+2)^2}}\right|$$

$$=\lim_{n\to\infty}\frac{n^2+4n+4}{n^2+2n+1}=1$$

the interval of convergence is $(-4,-2)$

8. Find the Taylor series of the function

$$f(x)=e^{2x}$$

at $x=0$, and give its radius and interval of convergence.(disregard the endpoints.)
[§11.5 #29]

$f(x)=e^{2x}$, $f'(x)=2e^{2x}$, $f''(x)=4e^{2x}$, ..., $f^{(n)}(x)=2^n e^{2x}$

$f(0)=1$, $f'(0)=2$, $f''(0)=4$, ..., $f^{(n)}(0)=2^n$

Therefore

$$f(x)=1+\frac{2}{1!}x+\frac{4}{2!}x^2+\dots+\frac{2^n}{n!}x^n+\dots$$

$$=\sum_{n=0}^\infty\frac{2^n}{n!}x^n$$

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\left|\frac{\frac{2^n}{n!}}{\frac{2^{n+1}}{(n+1)!}}\right|=\lim_{n\to\infty}\frac{n+1}{2}=\infty$$

the interval of convergence is $(-\infty,\infty)$