Calc. Homework Assignment-MGT9

Class: ____________________

Student Number: ____________________

Name: 芷雲

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1. Find the Taylor series of the function $f(x) = x^2e^{-x^2}$ at $x = 0$. Give the interval of convergence for the series. [§11.6 #11]

We replace x with $-x^2$ in the expression

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

to obtain

$$x^2e^{-x^2} = x^2\left[1 - x^2 + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \cdots + \frac{(-x^2)^n}{n!} + \cdots\right]$$

$$= x^2\left[1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \cdots\right]$$

$$= \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+2}}{n!} \quad ※$$

the interval of convergence for the series: $(-\infty, \infty)$ ※

2. Use a sixth-degree Taylor polynomial to approximate

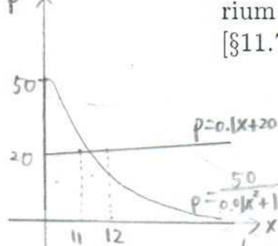
$$\int_0^{0.5}\frac{1}{\sqrt{1+x^2}}dx$$

[§11.6 #25]

Let $f(x) = \frac{1}{\sqrt{1+x^2}}$

$$f(x) \approx \sum_{n=0}^{6}\frac{f^{(n)}(0)x^n}{n!}$$

$$= 1 - \frac{1}{2}x^2 + \frac{3}{8}x^4 - \frac{5}{16}x^6$$



$$\int_0^{0.5}\frac{1}{\sqrt{1+x^2}}dx$$

$$\approx \int_0^{0.5}\left(1 - \frac{1}{2}x^2 + \frac{3}{8}x^4 - \frac{5}{16}x^6\right)dx$$

$$= x - \frac{1}{6}x^3 + \frac{3}{40}x^5 - \frac{5}{112}x^7\Big|_0^{0.5}$$

$$\approx 0.5 - 0.021 + 0.002 - 0$$

$$\approx 0.481 \quad ※$$ (更精確值亦正確)

3. Let $f(x) = 2x^3 - 9x^2 + 12x - 2$.

a. Show that $f(x) = 0$ has a root brtween $x = 0$ and $x = 1$.

b. Use Newton's method to find the zero of f in the interval $(0, 1)$, accurate to four decimal places. [§11.7 #19]

a. $\because f(0) = -2 < 0,\ f(1) = 3 > 0$

$f(0)\cdot f(1) < 0$ and f is continuous

$\therefore f(x) = 0$ has a root between $x = 0$ and $x = 1$ ※

b. $f(x) = 2x^3 - 9x^2 + 12x - 2$, $f'(x) = 6x^2 - 18x + 12$

Iterative formula: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = \frac{4x_n^3 - 9x_n^2 +}{6x_n^2 - 18x_n +}$

Guess $x_0 = 0$ (x_0可猜0與1之間的其它值)

$x_1 = \frac{2}{12} \approx 0.16667$

$x_2 = \frac{1.7685}{9.1661} \approx 0.19293$

$x_3 = \frac{1.6937}{8.7506} \approx 0.19355$.

$x_4 = \frac{1.6918}{8.7408} \approx 0.19355$

Ans: 0.1936 ※

4. The quantity of Sicard wristwatches demanded per month is related to the unit price by the equation

$$p = d(x) = \frac{50}{0.01x^2 + 1} \qquad (1 \le x \le 20)$$

where p is measured in dollars and x is measured in thousands. The supplier is willing to make x thousand wristwatches available per month when the price per watch is given by $p = s(x) = 0.1x + 20$ dollars. Find the equilibrium quantity and price. [§11.7 #39]

$$0.1x + 20 = \frac{50}{0.01x^2 + 1}$$

$$0.1x + 20 - \frac{50}{0.01x^2+1} = 0$$

Let $f(x) = 0.1x + 20 - \frac{50}{0.01x^2+1}$

$$f'(x) = 0.1 + 50(0.01x^2+1)^{-2}\cdot 0.02x$$

$$= 0.1 + \frac{x}{(0.01x^2+1)^2}$$

Iterative formula: $x_{n+1} = x_n - \frac{0.1x_n + 20 - \frac{50}{0.01x_n^2+1}}{0.1 + \frac{x_n}{(0.01x_n^2+1)^2}}$

Guess $x_0 = 11$ (x_0也可猜其它值)

$x_1 \approx 11.6481$

$x_2 \approx 11.6711$

$x_3 \approx 11.6711$

$x_4 \approx 11.6711$

$p = 0.1\cdot 11.6711 + 20 \doteq 21.17$

The equilibrium quantity is approximately 11671 wristwatches and price is approximately 21.17 dollars

5. Find the second derivative of the function

$$h(t) = (t^2+1)\sin t$$

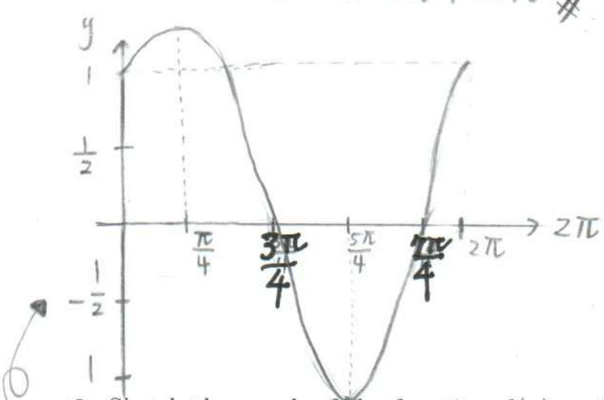
[§12.3 #33]

$h'(t) = 2t\sin t + (t^2+1)\cos t$

$h''(t) = 2\sin t + 2t\cos t + 2t\cos t - (t^2+1)\sin t$

$= 2\sin t + 4t\cos t - t^2\sin t - \sin t$

$= 4t\cos t - t^2\sin t + \sin t$ ※



6. Sketch the graph of the function $f(x) = \sin x + \cos x$ over the interval $0 \le x \le 2\pi$ by obtaining the following information:

a. The intervals where f is increasing and where it is decreasing

b. The relative extrema of f

c. The concavity of f

d. The inflection points of f

[§12.3 #47]

a. $f'(x) = \cos x - \sin x$ sign: $+$ on $(0, \frac{\pi}{4})$, $-$ on $(\frac{\pi}{4}, \frac{5\pi}{4})$, $+$ on $(\frac{5\pi}{4}, 2\pi)$

f is increasing on $(0, \frac{\pi}{4})$ and $(\frac{5\pi}{4}, 2\pi)$,

f is decreasing on $(\frac{\pi}{4}, \frac{5\pi}{4})$

b. $f(0) = 1$, $f(\frac{\pi}{4}) = \sqrt{2}$, $f(\frac{5\pi}{4}) = -\sqrt{2}$, $f(2\pi) = 1$

relative minimum : $-\sqrt{2}$ at $x = \frac{5\pi}{4}$

relative maximum: $\sqrt{2}$ at $x = \frac{\pi}{4}$

c. $f''(x) = -\sin x - \cos x$ sign: $-$ on $(0, \frac{3\pi}{4})$, $+$ on $(\frac{3\pi}{4}, \frac{7\pi}{4})$, $-$ on $(\frac{7\pi}{4}, 2\pi)$

f is concave upward on $(\frac{3\pi}{4}, \frac{7\pi}{4})$

f is concave downupward on $(0, \frac{3\pi}{4})$ and $(\frac{7\pi}{4}, 2\pi)$

d. 由 c 可知

inflection points : $(\frac{3\pi}{4}, 0), (\frac{7\pi}{4}, 0)$

7. Find the area of the region under the graph of the function $f(x) = \tan x$ from $x = 0$ to $x = \frac{\pi}{4}$.

[§12.4 #35]

$\int_0^{\frac{\pi}{4}} \tan x \, dx$

$= -\ln|\cos x| \Big|_0^{\frac{\pi}{4}}$

$= -\ln(\cos\frac{\pi}{4}) + \ln(\cos 0)$

$= -\ln\frac{\sqrt{2}}{2} + \ln 1$

$= (-\ln\sqrt{2} + \ln 2) + 0$

$= -\frac{1}{2}\ln 2 + \ln 2$.

$= \frac{1}{2}\ln 2$ ※

8. The weekly closing price of TMA Corporation stock in week t is approximated by

$$f(t) = 80 + 3t\cos\left(\frac{\pi t}{6}\right) \qquad (0 \le t \le 15)$$

where $f(t)$ is the price (in dollars) per share. Find the average weekly closing price of the stock over the 15-week period.

[§12.4 #43]

$A = \frac{1}{15-0}\int_0^{15} [80 + 3t\cos(\frac{\pi t}{6})]\,dt$

$= \frac{1}{15}\int_0^{15} 80\,dt + \frac{1}{15}\int_0^{15} 3t\cos(\frac{\pi t}{6})\,dt$

$= 80 + \frac{1}{5}\int_0^{15} t\cos(\frac{\pi t}{6})\,dt$

Let $u = t$, $dv = \cos(\frac{\pi t}{6})\,dt$

so that $du = dt$, $v = \frac{6}{\pi}\sin(\frac{\pi t}{6})$

$A = 80 + \frac{1}{5}\left[t \cdot \frac{6}{\pi}\sin(\frac{\pi t}{6})\Big|_0^{15} - \int_0^{15}\frac{6}{\pi}\sin(\frac{\pi t}{6})\,dt\right]$

$= 80 + \frac{1}{5}\left[\frac{90}{\pi}\sin(\frac{5\pi}{2}) + \frac{6}{\pi}\cdot\frac{6}{\pi}\cos(\frac{\pi t}{6})\Big|_0^{15}\right]$

$= 80 + \frac{1}{5}\left[\frac{90}{\pi} + (\frac{6}{\pi})^2(\cos\frac{5\pi}{2} - \cos 0)\right]$

$= 80 + \frac{1}{5}\left[\frac{90}{\pi} - (\frac{6}{\pi})^2\right]$

$\approx 80 + \frac{1}{5}(28.66 - 3.65) \approx 85$ (dollors per share