

Calculus Quiz 5 CSI-A

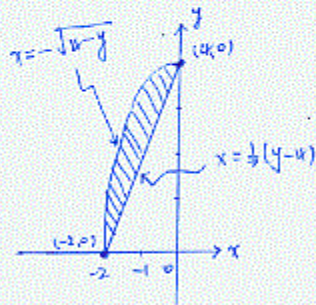
Class: 資工 1 A

Student Number: _____

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1. (5 points) Sketch the region of integration for the integral $\int_0^4 \int_{-\sqrt{4-y}}^{(y-4)/2} x \, dx \, dy$, reverse the order of integration, and evaluate the integral.



Since $x = \frac{1}{2}(y-4) \Rightarrow y = 2x+4$
 $x = -\sqrt{4-y} \Rightarrow y = 4-x^2$

$$\begin{aligned} \int_0^4 \int_{-\sqrt{4-y}}^{\frac{1}{2}(y-4)} x \, dx \, dy &= \int_{-2}^0 \int_{2x+4}^{4-x^2} x \, dy \, dx \\ &= \int_{-2}^0 \left(xy \Big|_{2x+4}^{4-x^2} \right) dx = \int_{-2}^0 (-x^3 - 2x^2) dx \\ &= -\frac{x^4}{4} - \frac{2}{3}x^3 \Big|_{-2}^0 = 0 - \left(-\frac{16}{3}\right) = \frac{16}{3} \end{aligned}$$

2. (5 points) Find the area of the region in the plane enclosed by the cardioid

$$r = 2(1 + \cos \theta).$$

$$\begin{aligned} A &= \frac{1}{2} \int_0^{2\pi} [2(1 + \cos \theta)]^2 d\theta \\ &= 2 \int_0^{2\pi} (1 + \cos^2 \theta + 2\cos \theta) d\theta \quad \text{by } \cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta \\ &= 2 \int_0^{2\pi} \left(1 + 2\cos \theta + \frac{1}{2} + \frac{1}{2} \cos 2\theta\right) d\theta \\ &= 2 \left\{ \frac{3}{2}\theta + 2\sin \theta + \frac{1}{4} \sin 2\theta \right\} \Big|_0^{2\pi} \\ &= 2 \{ 3\pi + 0 + 0 - 0 - 0 \} \\ &= 6\pi \end{aligned}$$

