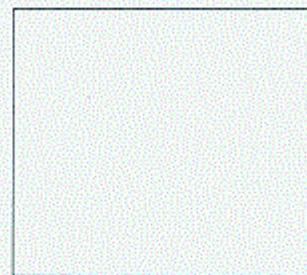


# Calculus Quiz 6 CSI-A

Class: 2<sup>nd</sup> I 1 A

Student Number: \_\_\_\_\_

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1. (5 points) Change the Cartesian integral  $\int_{-1}^0 \int_0^{\sqrt{1-y^2}} \frac{\sqrt{x^2+y^2}+1}{1+x^2+y^2} dx dy$  into an equivalent polar integral. Then evaluate the polar integral.

$$\begin{aligned} & \int_{-1}^0 \int_0^{\sqrt{1-y^2}} \frac{\sqrt{x^2+y^2}+1}{1+x^2+y^2} dx dy && \hat{=} \quad x = r \cos \theta, \quad y = r \sin \theta \\ & = \int_{\frac{3}{2}\pi}^{2\pi} \int_0^1 \frac{r+1}{1+r^2} \cdot r dr d\theta = \int_{\frac{3}{2}\pi}^{2\pi} \int_0^1 \frac{r^2}{1+r^2} dr d\theta + \int_{\frac{3}{2}\pi}^{2\pi} \int_0^1 \frac{r}{1+r^2} dr d\theta \\ & = \int_{\frac{3}{2}\pi}^{2\pi} \int_0^1 1 dr d\theta - \int_{\frac{3}{2}\pi}^{2\pi} \int_0^1 \frac{1}{1+r^2} dr d\theta + \int_{\frac{3}{2}\pi}^{2\pi} \int_0^1 \frac{r}{1+r^2} dr d\theta \\ & = \int_{\frac{3}{2}\pi}^{2\pi} 1 d\theta - \int_{\frac{3}{2}\pi}^{2\pi} \left[ \tan^{-1} r \right]_0^1 d\theta + \frac{1}{2} \int_{\frac{3}{2}\pi}^{2\pi} \int_0^1 \frac{1}{1+r^2} d(1+r^2) d\theta \\ & = \frac{1}{2} \pi - \frac{\pi}{4} \Big|_{\frac{3}{2}\pi}^{2\pi} + \frac{1}{2} \pi \ln 2 \Big|_{\frac{3}{2}\pi}^{2\pi} \\ & = \frac{\pi}{2} - \frac{\pi^2}{2} + \frac{2}{8} \pi^2 + \pi \ln 2 - \frac{3}{4} \pi \ln 2 = \frac{\pi}{2} - \frac{\pi^2}{8} + \frac{\pi}{4} \ln 2 \quad \# \end{aligned}$$

2. (5 points) Set up the limits of integration for evaluating the triple integral of a function  $F(x, y, z)$  over the tetrahedron  $D$  with vertices  $(0, 0, 0)$ ,  $(1, 1, 0)$ ,  $(0, 1, 0)$ , and  $(0, 1, 1)$ .

The integral is

$$\int_0^1 \int_0^{1-x} \int_{x+z}^1 F(x, y, z) dy dz dx$$

$$\left( \text{or } \int_0^1 \int_x^1 \int_0^{y-x} F(x, y, z) dz dy dx \right)$$