# Calculus Quiz 6 CSI-A

Class: 資工 1 A

Student Number:

Name: [illegible]

**1.** (*5 points*) Change the Cartesian integral $\int_{-1}^{0}\int_{0}^{\sqrt{1-y^2}} \frac{\sqrt{x^2+y^2}+1}{1+x^2+y^2}\,dx\,dy$ into an equivalent polar integral. Then evaluate the polar integral.

令 $x = r\cos\theta$, $y = r\sin\theta$

$$\int_{-1}^{0}\int_{0}^{\sqrt{1-y^2}} \frac{\sqrt{x^2+y^2}+1}{1+x^2+y^2}\,dx\,dy$$

$$= \int_{\frac{3}{2}\pi}^{2\pi}\int_0^1 \frac{r+1}{1+r^2}\cdot r\,dr\,d\theta = \int_{\frac{3}{2}\pi}^{2\pi}\int_0^1 \frac{r^2}{1+r^2}\,dr\,d\theta + \int_{\frac{3}{2}\pi}^{2\pi}\int_0^1 \frac{r}{1+r^2}\,dr\,d\theta$$

$$= \int_{\frac{3}{2}\pi}^{2\pi}\int_0^1 1\,dr\,d\theta - \int_{\frac{3}{2}\pi}^{2\pi}\int_0^1 \frac{1}{1+r^2}\,dr\,d\theta + \int_{\frac{3}{2}\pi}^{2\pi}\int_0^1 \frac{r}{1+r^2}\,dr\,d\theta$$

$$= \int_{\frac{3}{2}\pi}^{2\pi} 1\,d\theta - \int_{\frac{3}{2}\pi}^{2\pi}\left\{\tan^{-1} r\Big|_0^1\right\}d\theta + \frac{1}{2}\int_{\frac{3}{2}\pi}^{2\pi}\int_0^1 \frac{1}{1+r^2}\,d(1+r^2)\,d\theta$$

$$= \frac{1}{2}\pi - \frac{\pi}{4}\theta\Big|_{\frac{3}{2}\pi}^{2\pi} + \frac{1}{2}\theta\ln 2\Big|_{\frac{3}{2}\pi}^{2\pi}$$

$$= \frac{\pi}{2} - \frac{\pi^2}{2} + \frac{3}{8}\pi^2 + \pi\ln 2 - \frac{3}{4}\pi\ln 2 = \frac{\pi}{2} - \frac{\pi^2}{8} + \frac{\pi}{4}\ln 2$$

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**2.** (*5 points*) Set up the limits of integration for evaluating the triple integral of a function $F(x,y,z)$ over the tetrahedron $D$ with vertices $(0,0,0)$, $(1,1,0)$, $(0,1,0)$, and $(0,1,1)$.

The integral is

$$\int_0^1\int_0^{1-x}\int_{x+z}^{1} F(x,y,z)\,dy\,dz\,dx$$

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$$\left(\text{or} \quad \int_0^1\int_x^1\int_0^{y-x} F(x,y,z)\,dz\,dy\,dx\right)$$