Calculus Quiz 10 EARTH

Class: ______________________

Student Number: ______________________

Name: ______________________

1. (*5 points*) Writing definition for convergent sequence and applying the definition to show that $\lim_{n\to\infty} \frac{1}{n} = 0$.

(i) The sequence $\{a_n\}$ converges to L

$\Leftrightarrow \forall \varepsilon > 0, \exists N \in \mathbb{N}$

s.t. $\forall n, n \geq N \Rightarrow |a_n - L| < \varepsilon$

(ii) Given $\varepsilon > 0$,

choose $N = [\frac{1}{\varepsilon}] + 1 \in \mathbb{N}$

s.t. $\forall n, n \geq N = [\frac{1}{\varepsilon}] + 1$

$\Rightarrow |a_n - L| = |\frac{1}{n} - 0| = \frac{1}{n} \leq \frac{1}{N} = \frac{1}{[\frac{1}{\varepsilon}]+1} < \frac{1}{\frac{1}{\varepsilon}} = \varepsilon$

#

2. (*5 points*) Find the limit of each sequence $\{a_n\}$ if it converges.

a. $a_n = (-1)^n\left(1 - \frac{2}{n}\right)$.

b. $a_n = \frac{\cos^2 n}{3^n}$.

a. $\because \lim_{k\to\infty} a_{2k} = \lim_{k\to\infty} (-1)^{2k}(1 - \frac{2}{2k}) = 1$

$\lim_{k\to\infty} a_{2k+1} = \lim_{k\to\infty} (-1)^{2k+1}(1 - \frac{2}{2k+1}) = -1$

$\therefore \lim_{n\to\infty} a_n$ doesn't exist $\Rightarrow \{a_n\}$ is divergent.

b. $\because -1 \leq \cos n \leq 1 \Rightarrow 0 \leq \cos^2 n \leq 1$

$\therefore 0 \leq \frac{\cos^2 n}{3^n} \leq \frac{1}{3^n}$, and $\lim_{n\to\infty} 0 = 0 = \lim_{n\to\infty} \frac{1}{3^n}$

By Sandwich Thm: $\lim_{n\to\infty} \frac{\cos^2 n}{3^n} = 0$

Hence $\{a_n\}$ is convergent.