Calculus Quiz 8 EARTH

Class: ______________________

Student Number: ______________________

Name: ______________________

1. (*5 points*) Use the Green's Theorem area formula on page 1180 to find the area of the region enclosed by the ellipse $\mathbf{r}(t) = (b\cos t)\mathbf{i} + (a\sin t)\mathbf{j}$, $0 \le t \le 2\pi$.

$M = x = b\cos t \qquad dx = -b\sin t\,dt$

$N = y = a\sin t \qquad dy = a\cos t\,dt$

$$\text{Area} = \frac{1}{2}\oint_C x\,dy - y\,dx$$

$$= \frac{1}{2}\int_0^{2\pi} (ab\cos^2 t + ab\sin^2 t)\,dt$$

$$= \frac{1}{2}\int_0^{2\pi} ab\,dt = \pi ab.$$

2. (*5 points*) Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$.

Let $f(x,y,z) = x^2 + y^2 - z$, $\vec{p} = \vec{k}$

$\nabla f = 2x\vec{i} + 2y\vec{j} - \vec{k}$

$|\nabla f| = \sqrt{4x^2 + 4y^2 + 1}$

$|\nabla f \cdot \vec{p}| = |-1| = 1$

$$\therefore \text{Surface Area} = \iint_R \frac{|\nabla f|}{|\nabla f \cdot \vec{p}|}\,dA$$

$$= \iint_{x^2+y^2\le 4} \sqrt{4x^2+4y^2+1}\,dx\,dy = \int_0^{2\pi}\int_0^2 \sqrt{4r^2+1}\,r\,dr\,d\theta$$

$$= \int_0^{2\pi}\left[\frac{1}{12}(4r^2+1)^{3/2}\right]_0^2 d\theta = \int_0^{2\pi} \frac{1}{12}(17^{3/2} - 1)\,d\theta$$

$$= \frac{\pi}{6}(17\sqrt{17} - 1)$$

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