

Calculus Quiz 4 ENG-C

Class: _____

Student Number: _____

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1. (5 points) Find the local extreme values of the function

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4.$$

$$f_x = y - 2x - 2$$

$$f_y = x - 2y - 2$$

$$\Rightarrow (x, y) = (-2, -2)$$

$$f_{xx} = -2 < 0$$

$$f_{yy} = -2$$

$$f_{xy} = 1$$

$$f(-2, -2) = 8$$

is local maximum.

$$\Rightarrow f_{xx}f_{yy} - f_{xy}^2 > 0$$

2. (5 points) Find the points on the curve $xy^2 = 2$ nearest the origin.

$$f(x, y) = xy^2 - 2$$

$$g(x, y) = x^2 + y^2$$

~~$$\nabla g = \lambda \nabla f$$~~

$$\nabla f = \lambda \nabla g$$

$$\Rightarrow (y^2, 2xy) = \lambda (2x, 2y)$$

$$\Rightarrow \begin{cases} y^2 = \lambda 2x \\ 2xy = \lambda 2y \end{cases} \Rightarrow y^2 = 2x^2$$

$$\therefore 2x^3 - 2 = 0$$

$$\Rightarrow x = 1$$

$$\Rightarrow y = \pm\sqrt{2}$$

Ans: $(1, \sqrt{2}), (1, -\sqrt{2})$