

Calculus Quiz 2 SCI-C

Class: 理 C

Student Number: _____

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1. (5 points) Solve the initial value problem for \mathbf{r} as a vector function of t where

$$\frac{d\mathbf{r}}{dt} = \frac{3}{2}(2t+1)^{1/2}\mathbf{i} + e^{-3t}\mathbf{j} + \frac{1}{2t+1}\mathbf{k} \text{ and } \mathbf{r}(0) = \mathbf{k}.$$

$$\text{sol: } \mathbf{r} = \int \left[\frac{3}{2}(2t+1)^{1/2}\mathbf{i} + e^{-3t}\mathbf{j} + \frac{1}{2t+1}\mathbf{k} \right] dt$$

$$= \left(\sqrt{2}\left(t+\frac{1}{2}\right)^{3/2} \right)\mathbf{i} - \left(\frac{1}{3}e^{-3t} \right)\mathbf{j} + \left[\frac{1}{2}\ln\left|t+\frac{1}{2}\right| \right]\mathbf{k} + C \quad (1)$$

$$\Rightarrow \mathbf{r}(0) = \sqrt{2} \cdot \left(\frac{1}{2}\right)^{3/2}\mathbf{i} - \frac{1}{3}\mathbf{j} + \left(\frac{1}{2}\ln\frac{1}{2}\right)\mathbf{k} + C = \mathbf{k}$$

$$\Rightarrow C = \frac{1}{2}\mathbf{i} + \frac{1}{3}\mathbf{j} + \left(1 + \frac{1}{2}\ln 2\right)\mathbf{k} \quad (2)$$

$$\therefore \mathbf{r}(t) = \left[\sqrt{2}\left(t+\frac{1}{2}\right)^{3/2} - \frac{1}{2} \right]\mathbf{i} + \left(\frac{1}{3} - \frac{1}{3}e^{-3t} \right)\mathbf{j} + \left(\frac{1}{2}\ln\left|t+\frac{1}{2}\right| + 1 + \frac{1}{2}\ln 2 \right)\mathbf{k} \quad (2)$$

2. (5 points) Find \mathbf{T} , \mathbf{N} , and κ for the space curve

$$\mathbf{r}(t) = (\sqrt{2}e^t \cos t)\mathbf{i} + (\sqrt{2}e^t \sin t)\mathbf{j} + \sqrt{2}\mathbf{k}.$$

$$\text{sol: } \mathbf{v}(t) = \sqrt{2}(e^t \cos t - e^t \sin t)\mathbf{i} + \sqrt{2}(e^t \sin t + e^t \cos t)\mathbf{j} \quad (1)$$

$$\Rightarrow |\mathbf{v}(t)| = \sqrt{[\sqrt{2}(e^t \cos t - e^t \sin t)]^2 + [\sqrt{2}(e^t \sin t + e^t \cos t)]^2} = 2e^t \quad (1)$$

$$\Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\frac{\cos t - \sin t}{\sqrt{2}} \right)\mathbf{i} + \left(\frac{\sin t + \cos t}{\sqrt{2}} \right)\mathbf{j} \quad (1)$$

$$\Rightarrow \frac{d\mathbf{T}}{dt} = \left(\frac{-\sin t - \cos t}{\sqrt{2}} \right)\mathbf{i} + \left(\frac{\cos t - \sin t}{\sqrt{2}} \right)\mathbf{j}$$

$$\Rightarrow \left| \frac{d\mathbf{T}}{dt} \right| = \sqrt{\left(\frac{-\sin t - \cos t}{\sqrt{2}} \right)^2 + \left(\frac{\cos t - \sin t}{\sqrt{2}} \right)^2} = 1 \quad (1)$$

$$\Rightarrow \mathbf{N} = \frac{\frac{d\mathbf{T}}{dt}}{\left| \frac{d\mathbf{T}}{dt} \right|} = \left(\frac{-\sin t - \cos t}{\sqrt{2}} \right)\mathbf{i} + \left(\frac{\cos t - \sin t}{\sqrt{2}} \right)\mathbf{j}, \quad \kappa = \frac{1}{|\mathbf{v}|} \cdot \left| \frac{d\mathbf{T}}{dt} \right| = \frac{1}{2e^t} \quad (1)$$