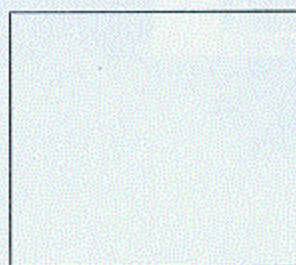


# Calculus Quiz 6 SCI-C

Class: 理 C

Student Number: \_\_\_\_\_

Name: 蘇偉霖



1. (5 points) Change the Cartesian integral  $\int_{-1}^0 \int_0^{\sqrt{1-y^2}} \frac{\sqrt{x^2+y^2}+1}{1+x^2+y^2} dx dy$  into an equivalent polar integral. Then evaluate the polar integral.

Sol: Let  $x=r\cos\theta$   $y=r\sin\theta$  (1)

$$\therefore \int_{\frac{3\pi}{2}}^{2\pi} \int_0^1 \frac{(r+1)}{1+r^2} \cdot r dr d\theta$$

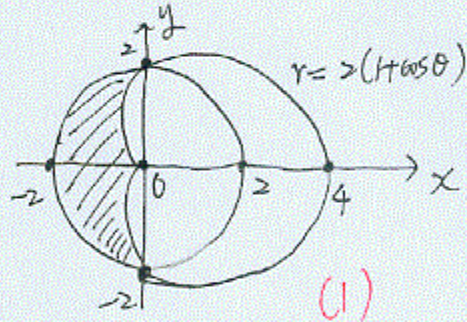
$$= \int_{\frac{3\pi}{2}}^{2\pi} \int_0^1 \frac{r^2}{1+r^2} dr d\theta + \int_{\frac{3\pi}{2}}^{2\pi} \int_0^1 \frac{r}{1+r^2} dr d\theta \quad (3)$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} \int_0^1 \left(1 - \frac{1}{1+r^2}\right) dr d\theta + \frac{1}{2} \int_{\frac{3\pi}{2}}^{2\pi} \int_0^1 \frac{2r}{1+r^2} dr d\theta$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} \left(1 - \frac{\pi}{4}\right) d\theta + \frac{1}{2} \int_{\frac{3\pi}{2}}^{2\pi} \ln 2 d\theta = \frac{1}{2}\pi - \frac{\pi^2}{8} + \frac{\pi}{4} \ln 2 \quad (1)$$

2. (5 points) Find the area of the region inside the circle  $r=2$  and outside the cardioid  $r=2(1+\cos\theta)$ .

Sol:



$$\text{area} = 2 \cdot \int_{\frac{3\pi}{2}}^{\pi} \int_{2(1+\cos\theta)}^2 r dr d\theta \quad (1)$$

$$= -4 \int_{\frac{3\pi}{2}}^{\pi} \cos^2\theta + 2\cos\theta d\theta \quad (1)$$

$$= -4 \left[ \frac{\theta}{2} + \frac{\sin 2\theta}{4} + 2\sin\theta \right]_{\frac{3\pi}{2}}^{\pi} \quad (1)$$

$$= 8\pi \quad (1)$$