

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If  $f(x) = \begin{cases} 1, & \text{if } x \text{ is rational.} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$  Then  $\lim_{x \rightarrow 0} xf(x)$  does not exist.
2. If  $f'(a)$  does not exist, then it is still possible that  $f$  is continuous at  $a$ .
3.  $\frac{d}{dx}|x^2 + x| = |2x + 1|$ .
4. If both  $f$  and  $g$  are differentiable at  $x = 0$  but  $g'(0) = 0$  then  $\frac{f(x)}{g(x)}$  is not differentiable at  $x = 0$ .
5. If  $f(x)$  is continuous and has a root in  $[a, b]$ , then  $f(a)$  and  $f(b)$  have opposite signs.
6. If  $f$  is continuous at 5,  $f(5) = 2$  and  $f(4) = 3$ , then  $\lim_{x \rightarrow 2} f(4x^2 - 11) = 2$ .
7. We can obtain the graph of  $y = x^2 + 6x + 10$  by shifting the graph of  $y = x^2$  3 units to the right and then 1 unit upward.
8. If  $f(x) = g(x)$  when  $x \neq 0$ , then  $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} g(x)$ , provided the limits exist.
9. If  $f(x) > g(x)$  for all  $x$  near  $a$  (except possibly at  $a$ ), and both  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} g(x)$  exist, then  $\lim_{x \rightarrow a} f(x) > \lim_{x \rightarrow a} g(x)$ .
10. If  $f(x)$  is an even function and  $f'(3) = 2$ , then  $f'(-3) = -2$ .

(下頁還有試題)

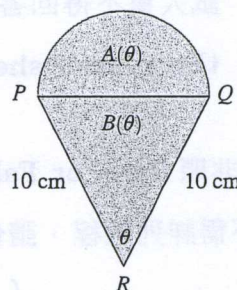
填充題 Short answer questions (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. A semicircle with diameter  $PQ$  sits on an isosceles triangle  $PQR$  to form a region shaped like a two-dimensional icecream cone, as shown in the figure.

If  $A(\theta)$  is the area of the semicircle and  $B(\theta)$  is the area of the triangle, find

$\lim_{\theta \rightarrow 0^+} \frac{A(\theta)}{B(\theta)}$ . Answer : \_\_\_\_\_.



2. If  $f'(0) = -1$ , find  $\lim_{h \rightarrow 0} \frac{f(3h) - f(-2h)}{h}$ . Answer : \_\_\_\_\_.
3. Find numbers  $a$  and  $b$  such that  $\lim_{x \rightarrow 0} \frac{\sqrt{ax+b} - 2}{x} = 1$ . Answer : \_\_\_\_\_.
4. If  $g(x) = \sqrt{2-x}$ , find the domain of the composite function  $g \circ g$ .  
Answer : \_\_\_\_\_.
5. If  $\lim_{x \rightarrow -3} \frac{f(x)}{x^2} = 1$ , find  $\lim_{x \rightarrow -3} \left( f(x) + \frac{f(x)}{x} \right)$ . Answer : \_\_\_\_\_.
6. Determine  $\lim_{x \rightarrow \frac{1}{2}^-} \frac{2x^2 - x}{4x^2 - 4x + 1}$ . Answer : \_\_\_\_\_.
7. Find all the vertical asymptotes of  $f(x) = \frac{\tan(\pi x)}{x(x-2)}$ . Answer : \_\_\_\_\_.
8. Let  $F(x) = f(xf(x^2))$ . If  $f(1) = -1$ ,  $f'(1) = 3$ ,  $f(-1) = 2$ , and  $f'(-1) = 4$ , find  $F'(1)$ . Answer : \_\_\_\_\_.

(下頁還有試題)

