

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If f is continuous at $x = a$, then f is differentiable at $x = a$.
2. If $\lim_{x \rightarrow 3^+} f(x)$ and $\lim_{x \rightarrow 3^-} f(x)$ both exist, then $\lim_{x \rightarrow 3} f(x)$ exists.
3. Let $F(x) = f(f(x))$. Then $F'(x) = [f'(x)]^2$.
4. If f is differentiable, then $\frac{d}{dx}[\sqrt{f(x)}] = \frac{f'(x)}{2\sqrt{f(x)}}$.
5. If C is a differentiable total cost function, then the marginal average cost function is $\overline{C}'(x) = \frac{C(x) - xC'(x)}{x^2}$.
6. If $\lim_{x \rightarrow a} f(x) = L$ and $g(x) = M$, then $\lim_{x \rightarrow a} f(x)g(x) = LM$.
7. $\lim_{x \rightarrow 1} \left(\frac{2x}{x-1} - \frac{2}{x-1} \right) = \lim_{x \rightarrow 1} \frac{2x}{x-1} - \lim_{x \rightarrow 1} \frac{2}{x-1}$.
8. $\lim_{x \rightarrow -2^-} \frac{x-3}{x+2} = \infty$
9. $\lim_{x \rightarrow \infty} \frac{-2x^3 - 3x^2 + 1}{x^2 + 2x + 5} = -\infty$
10. Suppose the function f is continuous on the interval $[a, b]$. If $f(a) \cdot f(b) > 0$, then f has no zero on $[a, b]$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$. Find the domain of the composite function $g \circ f$. Answer : _____.
2. Suppose $F(x) = f(x^2 + 1)$ and $f'(2) = 3$. Find $F'(1)$.
Answer : _____.
3. Suppose $h = f \circ g$ and $f(0) = 2$, $f'(4) = -3$, $g(0) = 4$, $g'(0) = 5$. Find $h'(0)$.
Answer : _____.
4. Evaluate $\lim_{x \rightarrow 3} \frac{x\sqrt{x^2 + 7}}{2x - \sqrt{2x + 3}}$. Answer : _____.
5. Evaluate $\lim_{t \rightarrow 0} \frac{1 - (1 + t)^2}{t(1 + t)^2}$. Answer : _____.
6. Let $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$. Find the equation of the tangent line through $(4, \frac{5}{2})$.
Answer : _____.
7. Evaluate $\lim_{h \rightarrow 0} \frac{\sqrt[3]{1+h} - 1}{h}$. Answer : _____.
8. During the construction of a high-rise building, a worker accidentally dropped his portable electric screwdriver from a height of 144 ft. After t sec, the screwdriver had fallen a distance of $s = 16t^2$ ft. What was the velocity of the screwdriver at the time it hit the ground? Answer : _____.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find A and B such that the function $f(x) = \begin{cases} x^2, & \text{if } x \leq 1 \\ Ax + B, & \text{if } x > 1 \end{cases}$ is differentiable at $x = 1$.

(下頁還有試題)

2. (10 points) Find the derivatives of the following functions.
- $f(x) = \frac{\sqrt{x+1}}{\sqrt{x^2+1}}$.
 - $g(x) = \left(\frac{x^2+3}{12x}\right) \left(\frac{x^4-1}{x^3}\right)$.
3. (10 points) Let $f(x) = \frac{1}{\sqrt{x}}$.
- Find and simplify the form $\frac{f(a+h) - f(a)}{h}$.
 - Use (a.) to find $f'(2)$.
4. (10 points) An apple orchard has an average yield of 36 bushels of apples/tree if tree density is 22 trees/acre. For each unit increase in tree density, the yield decreases by 2 bushels/tree. Letting x denote the number of trees beyond 22/acre, find a function in x that gives the yield of apple.
5. (10 points) Mary works at the B&D department store, where, on a weekday, she is paid \$8 an hour for the first 8 hours and \$12 an hour for overtime.
- Find a function f that gives Mary's earning on a weekday in which she worked x hours.
 - Can you determine the rate of change of Mary's earning at $x = 8$? Why?
6. (10 points) The demand function for a certain make of exercise bicycle sold exclusively through cable television is

$$p = \sqrt{9 - 0.02x} \quad (0 \leq x \leq 450)$$

where p is the unit price in hundreds of dollars and x is the quantity demanded/week. Compute the elasticity of demand and determine the range of prices corresponding to inelastic, unitary, and elastic demand.

Hint: Solve the equation $E(p) = 1$.

(試題結束)