

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $f(x)$ is continuous and $\int_1^3 f(x) dx = 8$, then f takes on the value 4 at least once on the interval $[1, 3]$.

2. If $f'(c) = 0$, then f has local maximum or minimum at c .

3. If f has a local maximum at c , then $f'(c) = 0$.

4. $\frac{d}{dx} \int_2^{x^3} t^9 dt = \frac{d}{dx} \int_1^{x^3} t^9 dt$.

5. If f is continuous on $[a, b]$, then $\frac{d}{dx} \left(\int_a^b f(x) dx \right) = f(x)$.

6. If f is continuous on $[0, 1]$, then $\int_0^1 f(x) dx = \int_0^1 f(1-x) dx$.

7. Let $f(t) = \int_{\sqrt{t}}^{t^2} \frac{\sqrt{1+u^4}}{u} du$ for $t > 0$ and $F(x) = \int_1^x f(t) dt$ for $x > 0$. Then $F(x)$ has a local minimum at $x = 1$.

8. If f and g are continuous on $[a, b]$, then

$$\int_a^b f(x) \cdot g(x) dx = \int_a^b f(x) dx \cdot \int_a^b g(x) dx.$$

9. The graph of every polynomial of odd degree has at least one horizontal tangent.

10. The function $g(x) = x|x|$ has an inflection point at $(0, 0)$. Hence $g''(0) = 0$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the value of $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x + 1} + x)$.

Answer : _____.

2. Find the limit $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \dots + \frac{1}{\sqrt{n}\sqrt{n+n}} \right)$.

Answer : _____.

3. Find the absolute extremum values of $f(x) = 3 \sin x + \sin 3x$ on $[0, 2\pi]$.

Answer : _____.

4. Evaluate $\int_1^2 x\sqrt{x-1} dx$. Answer : _____.

5. Find the area of the region enclosed by the given curves $y = x^2$ and $y = 4x - x^2$.

Answer : _____.

6. Two cars start moving from the same point. One travels south at 30 km/h and the other travels east at 40 km/h. At what rate is the distance between the two cars increasing two hours later? Answer : _____.

7. Find the linearization of $h(x) = 3 + \int_1^{x^2} \sqrt{3+r^3} dr$ at $x = -1$.

Answer : _____.

8. If f is a continuous function such that

$$\int_0^x f(t) dt = x \sin x + \int_0^x \frac{f(t)}{2+t^2} dt$$

for all x , find an explicit formula for $f(x)$. Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find the volume of the solid obtained by rotating about the x -axis the region bounded by $x = 1 + y^2$, $x = 0$, $y = 1$ and $y = 2$.

2. (10 points)

a. Please describe the Mean Value Theorem.

b. Use the Mean Value Theorem to prove the inequality

$$|\sin x - \sin y| \leq |x - y|$$

for all x and y .

3. (10 points) Let $f(x) = x^{1/4}(2 - x)^{3/4}$

a. (7 points) Find the absolute maximum value of $f(x)$ on $[0, 1]$.

b. (3 points) Show that $0 \leq \int_0^1 f(x) \cos^2 x \, dx \leq \frac{3}{2}$.

4. (10 points) Find the area of the largest isosceles(等腰) triangle that can be inscribed in a circle of radius r .

5. (10 points) Let $f(x) = \frac{x^2 + 4}{x}$. Find **a.** its domain, **b.** critical numbers, **c.** intervals of increasing/decreasing, **d.** local maximum/minimum values, **e.** concavity, **f.** inflection points, **g.** slant, horizontal and vertical asymptotes. **h.** Then sketch the graph of $f(x)$.

6. (10 points)

a. State the Fundamental Theorem of Calculus Part I and Part II.

b. Find $\frac{d}{dx} \int_0^{\sin x} \cos t \, dt$.

(試題結束)