

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $f(x)$ is a polynomial of degree n , then $f^{(n+1)}(x) = 0$.
2. If $h''(x) < 0$ on (a, b) and $h'(c) = 0$ where $a < c < b$, then $h(c)$ is the absolute maximum value of h on $[a, b]$.
3. $\ln x^3 = 3 \cdot \ln x$ for all x in $[0, \infty)$.
4. $e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x \approx 2.718\dots$
5. $\ln \frac{x^2 \sqrt{x^2 - 1}}{e^x} = 2 \ln x + \frac{1}{2} \ln(x - 1) + \frac{1}{2} \ln(x + 1) - x$, for all $x < -1$.
6. Let $A =$ Accumulated amount at the end of t years
 $P =$ Principal
 $r =$ Nominal interest rate per year
 $m =$ Number of conversion periods per year
 $t =$ Term (number of years)
 Then the compound interest of t years is $A = P \left(1 + \frac{r}{m}\right)^{mt}$.
7. The line $y = a$ is a horizontal asymptote of the graph of a function f if either
 $\lim_{x \rightarrow a^+} f(x) = \pm\infty$ or $\lim_{x \rightarrow a^-} f(x) = \pm\infty$.
8. If $f''(x_0) = 0$, then $(x_0, f(x_0))$ is an inflection point of the graph of f .
9. If $0 < b < 1$ and $x < y$, then $b^x > b^y$.

10. If f is continuous on the closed interval $[a, b]$, then f has an absolute minimum value.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the present value of \$40,000 due in 4 years at 3%/year compounded continuously interest rate. (列式即可) Answer : _____.
2. If $49^x + 7^x - 72 = 0$, find x . Answer : _____.
3. If $s = f(t)$ gives the position of an object moving on the coordinate line, what do $f'(t)$ and $f''(t)$ measure? Answer : _____.
4. Let a and b be the absolute maximum and absolute minimum values of the function $f(x) = x^{2/3}$ on the interval $[-1, 8]$. Find $a+b$. Answer : _____.
5. Find an equation of the tangent line to the graph of the function f defined by the equation $x^2y^3 - y^2 + xy - 1 = 0$ at the point $(1, 1)$. Answer : _____.
6. A 6-ft tall man is walking away from a street light 18 ft high at a speed of 6 ft/sec. How fast is the tip of his shadow moving along the ground?
Answer : _____.
7. Let $f(x) = x - 2\sqrt{x}$. Find the absolute extrema of f . Answer : _____.
8. Effective rate $\hat{r}_{\text{eff}} = \underline{\hspace{2cm}}^r - 1$, where r is a nominal interest rate per year compounded continuously.(填空) Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) An apple orchard has an average yield of 36 bushels of apples/tree if tree density is 22 trees/acre. For each unit increases (decreases) in tree density, the yield decreases (increases) by 2 bushels/tree. How many trees should be planted in order to maximize the yield?
2. (10 points) A developing country's gross domestic product (GDP) from 2000 to 2008 is approximated by the function

$$G(t) = -0.2t^3 + 2.4t^2 + 60 \quad (0 \leq t \leq 8)$$

where $G(t)$ is measured in billions of dollars, with $t = 0$ corresponding to 2000. Sketch the graph of the function G and find the absolute extrema and inflection points.

3. (10 points) [Packaging] Betty Moore Company requires that its corned beef hash containers have a capacity of 54 cubic inches, have the shape of right circular cylinders, and be made of aluminum. Determine the radius and height of the container that requires the least amount of metal.
4. (10 points) The supply equation for a certain brand of radio is given by

$$p = s(x) = 0.3\sqrt{x} + 10$$

where x is the quantity supplied and p is the unit price in dollars. Use differentials to approximate the change in price when the quantity supplied is increased from 10,000 units to 10,500 units.

(下頁還有試題)

5. (10 points) The function

$$I(t) = -0.2t^3 + 3t^2 + 100 \quad (0 \leq t \leq 9)$$

gives the CPI of an economy, where $t = 0$ corresponds to the beginning of 2002.

- a. Find the inflation rate at the beginning of 2008 ($t = 6$).
 - b. Show that inflation was moderating at that time.
6. (10 points) How long will it take \$10,000 to grow to \$15,000 if the investment earns an interest rate of 12% per year compounded quarterly?
($\ln 1.5 = 0.4055$, $\ln 1.03 = 0.0296$)

(試題結束)