

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $f(x) = g(x)$ when $x \neq 0$, then $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} g(x)$, provided the limits exist.
2. If $f'(c) = 0$, then f has local maximum or minimum at c .
3. If f is continuous on $[0, 1]$, then $\int_0^1 f(x) dx = \int_0^1 f(1-x) dx$.
4. If f is increasing and concave upward, then its inverse function is concave downward.
5. $\frac{d}{dx} 10^x = x 10^{x-1}$.
6. $\cos(\tan^{-1} x) = \frac{1}{\sqrt{1+x^2}}$.
7. $\int_0^4 \frac{x}{x^2-1} dx = \frac{1}{2} \ln 15$.
8. $\lim_{x \rightarrow \pi^-} \frac{\tan x}{1 - \cos x} = \lim_{x \rightarrow \pi^-} \frac{\sec^2 x}{\sin x} = \infty$.
9. If f is continuous on $[0, \infty)$ and $\int_1^\infty f(x) dx$ is convergent, then $\int_0^\infty f(x) dx$ is convergent.
10. If f is continuous, then $\int_{-\infty}^\infty f(x) dx = \lim_{t \rightarrow \infty} \int_{-t}^t f(x) dx$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the area of the region enclosed by the given curves $y = x^2$ and $y = 4x - x^2$.

Answer : _____.

2. Find the derivative of $y = (\cos x)^{\sin x}$.

Answer : _____.

3. Evaluate $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \sin(t^2) dt$.

Answer : _____.

4. Evaluate $\int \sqrt{\frac{1-x}{1+x}} dx$.

Answer : _____.

5. The curve $y = \sqrt{4-x^2}$, $-1 \leq x \leq 1$, is an arc of the circle $x^2 + y^2 = 4$. Find the area of the surface obtained by rotating this arc about x -axis.

Answer : _____.

6. Use logarithmic differentiation to find $\frac{d}{dx} \sqrt[3]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$.

Answer : _____.

7. Calculate $g' \left(\frac{\pi}{4} \right)$, where g is the inverse function of $f(x) = \ln x + \tan^{-1} x$.

Answer : _____.

8. Evaluate $\int_0^1 \ln x dx$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find $\frac{dy}{dx}$ of the curve

$$y \cos x = 1 + \sin(xy)$$

by using implicit differentiation.

2. (10 points) Evaluate the integral

$$\int \ln(x^2 - x + 2) dx.$$

3. (10 points) Evaluate

$$\int \frac{x}{\sqrt{3 - 2x - x^2}} dx.$$

4. (10 points) Find the length of the arc of the parabola $y^2 = x$ from $(0, 0)$ to $(1, 1)$.

(Hint: $\int \sec^3 \theta d\theta = \frac{1}{2}(\sec \theta \tan \theta + \ln |\sec \theta + \tan \theta|) + c$)

5. (10 points) Determine whether

$$\int_0^\pi \frac{\sin^2 x}{\sqrt{x}} dx$$

is convergent or divergent.

6. (10 points) For what value of a is the following equation true?

$$\lim_{x \rightarrow \infty} \left(\frac{x+a}{x-a} \right)^x = e$$

(試題結束)