

考試時間 120 分鐘，題目卷為三張紙，共 6 頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If f is continuous at $x = a$, then f is differentiable at $x = a$.
2. If $h''(x) < 0$ on (a, b) and $h'(c) = 0$ where $a < c < b$, then $h(c)$ is the absolute maximum value of h on $[a, b]$.
3. If f is differentiable, then $\frac{d}{dx}[\sqrt{f(x)}] = \frac{f'(x)}{2\sqrt{f(x)}}$.
4. If $\lim_{x \rightarrow a} f(x) = L$ and $g(x) = M$, then $\lim_{x \rightarrow a} f(x)g(x) = LM$.
5. If f is continuous on the closed interval $[a, b]$, then f has an absolute minimum value.
6. If $f(x) = \ln 7$, then $f'(x) = \frac{1}{7}$.
7. The amount of an annuity is

$$A = \frac{mP}{r}(e^{rT} - 1)$$

where P = size of each payment in the annuity, r = interest rate compounded continuously, T = term of the annuity (in years) and m = number of payments per year.

8. $\int_0^1 (1-x)^4 dx = \int_0^1 x^4 dx$.

9. If $f(x)$ and $g(x)$ are continuous on $[a, b]$, then the area of the region bounded by the graphs of $f(x)$ and $g(x)$ and the vertical lines $x = a$ and $x = b$ is given by $\int_a^b [f(x) - g(x)]dx$.
10. $\int_{-1}^1 \frac{1}{x} dx = \ln|x| \Big|_{-1}^1 = \ln|1| - \ln|-1| = \ln 1 - \ln 1 = 0$.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

- Evaluate $\lim_{h \rightarrow 0} \frac{\sqrt[3]{1+h} - 1}{h}$. Answer : _____.
- Find an equation of the tangent line to the graph of the function f defined by the equation $x^2y^3 - y^2 + xy - 1 = 0$ at the point $(1, 1)$. Answer : _____.
- Let $f(x) = x^{\ln x}$. Find $f'(e)$. Answer : _____.
- Wood deposits recovered from an archeological site contain 20% of the C-14 they originally contained. How long ago did the tree from which the wood was obtained die?(Hint: The decay constant of C-14 is 0.00012.)(列式即可)
Answer : _____.
- Find the area of the region R that is completely enclosed by the graphs of the functions $f(x) = 2x - 1$ and $g(x) = x^2 - 4$. Answer : _____.
- Evaluate $\int x2^x dx$. Answer : _____.
- Suppose f'' is continuous on $[1, 3]$ and $f(1) = 2, f(3) = -1, f'(1) = 2$, and $f'(3) = -1$. Evaluate $\int_1^3 x f''(x) dx$. Answer : _____.
- Crystal Car Wash recently bought an automatic car-washing machine that is expected to generate \$40,000 in revenue per year, t years from now, for the next 5 years. If the income is reinvested in a business earning interest at the rate of 4% per year compounded continuously, find the total accumulated value of this income stream at the end of 5 years. (Hint: $e^{0.2} \approx 1.22$) Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Evaluate the following integrals.

a. $\int \frac{\ln x}{\sqrt{x}} dx$

b. $\int_0^2 r\sqrt{5 - \sqrt{4 - r^2}} dr$

2. (10 points) Find an equation of the tangent line to the graph of $y = xe^{-x}$ at the point $(1, e^{-1})$.

3. (10 points) In a study conducted by a certain country's Economic Development Board, it was found that the Lorentz curve for the distribution of income of stockbrokers was described by the function

$$f(x) = \frac{11}{12}x^2 + \frac{1}{12}x$$

and that of high school teachers by the function

$$g(x) = \frac{5}{6}x^2 + \frac{1}{6}x.$$

a. Compute the coefficient of inequality for each Lorentz curve.

b. Which profession has a more equitable income distribution?

4. (10 points) Tracy purchased a 15-yr franchise for a computer outlet store that is expected to generate income at the rate of $P(t) = 50,000 + 3000t$ dollars/year. If the prevailing interest rate is 10%/year compounded continuously, find the present value of the franchise.

(下頁還有試題)

5. (10 points) Find the area of the region bounded by the graph of the function $f(x) = \frac{x+1}{\sqrt{x}}$, the x -axis, and the lines $x = a$ and $x = b$ where a and b are, respectively, the x -coordinates of the relative minimum point and inflection point of f .

6. (10 points) The number of voters in a certain district of a city is expected to grow at the rate of

$$R(t) = \frac{3000}{\sqrt{4+t^2}}$$

people/year, t yr from now. If the number of voters at present is 20,000, how many voters will be in the district 5 yr from now?

(試題結束)