微積分聯合教學　段考五　　May 8, 2012

考試時間 120 分鐘，題目卷爲兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名；題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材(包含手機)，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The equation $3x^2+3y^2+3z^2=10+6y+12z$ represents a sphere with radius 5.

2. There are scalars s and t such that

$$\langle 7,1\rangle = s\langle 2,3\rangle + t\langle -2,1\rangle.$$

3. The tangent line to the curve $x=e^{-t}\cos t$, $y=e^{-t}\sin t$, $z=e^{-t}$ at the point $(1,0,1)$ is a line parallel to the vector $\langle -1,1,-1\rangle$.

4. Let $\mathbf{r}(t)=2\cos t\mathbf{i}+\sin t\mathbf{j}+2t\mathbf{k}$, then $\displaystyle\int_0^{\pi/2}\mathbf{r}(t)\,dt = 3+\frac{\pi^2}{4}$.

5. Let C be the circle traced by the vector function $\mathbf{r}(t)=\langle\cos 2t,\sin 2t\rangle$, then the circumference of C is $\displaystyle\int_0^{2\pi}|\mathbf{r}'(t)|\,dt$.

In problems 6 and 7, consider the two variable function $F(x,y)=1+\sqrt{4-y^2}$.

6. The domain of F is the closed interval $[-2,2]$.

7. The range of F is the closed interval $[1,3]$.

In problems 8 and 9, consider the following two varible function

$$f(x,y)=\begin{cases}\dfrac{x^2y^3}{2x^2+y^2}, & (x,y)\neq(0,0)\\ 1, & (x,y)=(0,0)\end{cases}$$

8. $\lim_{(x,y)\to(0,0)} f(x,y)$ exists.

9. $f(x,y)$ is continuous at $(0,0)$.

10. The directional derivative of a differentiable real-valued function $f(x,y,z)$ in the direction of a *unit* vector $\mathbf{u} = \langle a, b, c \rangle$ is

$$D_u(x,y,z) = a\frac{\partial f}{\partial x} + b\frac{\partial f}{\partial y} + c\frac{\partial f}{\partial z}.$$

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{j} - 2\mathbf{k}$, find $|\mathbf{a} - \mathbf{b}|$. $\boxed{\text{A}}$

Answer ： ____________.

2. Find the component $z(t)$ of the vector function $\mathbf{r}(t) = x(t)\mathbf{i} + t\mathbf{j} + z(t)\mathbf{k}$ that represents the curve of intersection of the hyperboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$. $\boxed{\text{B}}$

Answer ： ____________.

3. Match the parametric equations with the graphs (labeled I–VI). One point for each correct match.

3.1 $x = \cos 4t,\ y = t,\ z = \sin 4t.$ $\boxed{\text{C}}$

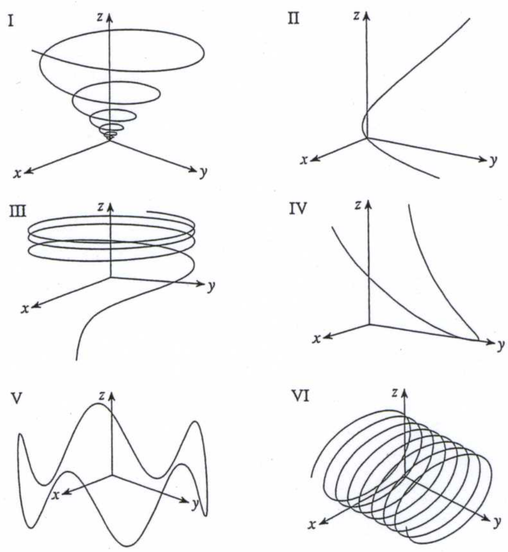
3.2 $x = t,\ y = t^2,\ z = e^{-t}.$ $\boxed{\text{C}}$

3.3 $x = t,\ y = 1/(1+t^2),\ z = t^2.$ $\boxed{\text{C}}$

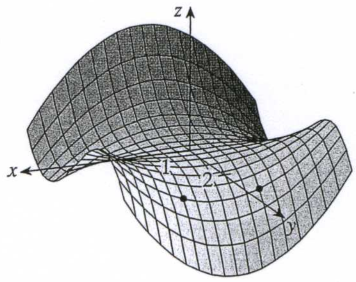
3.4 $x = e^{-t}\cos 10t,\ y = e^{-t}\sin 10t,\ z = e^{-t}.$ $\boxed{\text{C}}$

3.5 $x = \cos t,\ y = \sin t,\ z = \sin 5t.$ $\boxed{\text{C}}$

Answer ： ____________.



4. Determine the signs (+ for positive, − for negative) of the partial derivatives for the function f whose graph is shown below. One point for each correct sign.



4.1 $f_x(1,2)$. [D] **4.2** $f_y(1,2)$. [D]

4.3 $f_{xx}(1,2)$. [D] **4.4** $f_{yy}(1,2)$. [D]

4.5 $f_{xy}(1,2)$. [D]

Answer : ___________.

5. Find the linearization $L(x,y)$ of $f(x,y) = xe^{xy}$ at the point $(1,0)$. [E]

Answer : ___________.

6. Find $\dfrac{dy}{dx}$ if $x^3 + y^3 = 6xy$. [F]

Answer : ___________.

7. Find the equation of the tangent plane at the point $(-2,1,-3)$ to the ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3. \quad \boxed{G}$$

Answer : ___________.

8. Find the value of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{(-1)^n}{2n+1} + \cdots$. [Hint: Recall the Maclaurin series of $\tan^{-1} x$. [H]

Answer : ___________.

(下頁還有試題)