

考試時間 120 分鐘，題目卷為三張紙，共 6 頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The function $f(x) = \frac{1}{4}e^{3x} + Ce^{-x}$ is a solution of the differential equation $y' + y = 2e^{3x}$.

2. The total differential of $w = f(x, y, z) = x\sqrt{y} + y\sqrt{z}$ is

$$dw = \sqrt{y}dx + \left(\frac{x}{2\sqrt{y}} + \sqrt{z}\right)dy + \frac{2y}{\sqrt{z}}dz.$$

3. Let R be the plane region bounded by the graphs of $y = x^2$ and $y = x$. Then

$$\iint_R xe^y dA = \int_0^1 \int_y^{\sqrt{y}} xe^y dx dy.$$

4. The population density of a coastal town is described by the function

$$f(x, y) = \frac{10,000e^y}{1 + 0.5|x|}, \quad -10 \leq x \leq 10, \quad -4 \leq y \leq 0, \quad \text{where } x \text{ and } y \text{ are measured}$$

in miles. Then the population of this town is $2 \int_{-10}^0 \int_{-4}^0 \frac{10,000e^y}{1 + 0.5x} dy dx$.

5. The differential equation $y' = xy + 2x - y - 2$ is separable.

6. If $\int_a^b f(x)dx = 1$, then f is a probability density function on $[a, b]$.

7. If f is a probability density function of a continuous random variable X in the interval $[a, b]$, then $\int_a^b x^2 f(x)dx = \text{Var}(X) + \left[\int_a^b x f(x)dx\right]^2$.

8. The function $f(x) = \frac{7}{8x^2}$ is a probability density function on the interval $[1, 8]$.

(下頁還有試題)

9. Let Z be the standard normal random variable. Then

$$P(0 \leq Z < 1.25) = \frac{1}{2} - P(Z < -1.25).$$

10. If $f(x) = \frac{1}{9}xe^{-x/3}$ is the probability density function for the random variable X on $[0, \infty)$, then $Var(X) = -1$.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let $z = f(x, y) = \ln(xy)^{1/2}$. Find the approximate change in z when the point changes from $(5, 10)$ to $(5.04, 9.94)$. Answer : _____.

2. Evaluate the double integral $\iint_R f(x, y)dA$ for the function $f(x, y) = xe^{y^2}$ and the region R which is bounded by $x = 0$, $y = x^2$, and $y = 4$.

Answer : _____.

3. Find the volume of the solid bounded above by the surface $z = f(x, y) = \frac{2y}{1+x^2}$ and below by the plane region R which is bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$.

Answer : _____.

4. Let $S(t)$ denote the supply of a certain commodity as a function of time t . Suppose the rate of change of the supply is proportional to the difference between the demand $D(t)$ and the supply. Find a differential equation that describes this situation. Answer : _____.

5. Solve the first-order differential equation $y' = \frac{xy^2}{\sqrt{1+x^2}}$ by separating variables.

Answer : _____.

6. Use Euler's method with $n = 2$ to obtain an approximation of the initial value problem $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1$ when $x = 1.2$. Round your answer to four decimal places, if necessary. Answer : _____.

7. A random variable X is uniformly distributed over $[0, 1]$ with probability density function $f(x) = 1$. Find $Var(X)$. Answer : _____.

(下頁還有試題)

8. According to data released by the Chamber of Commerce of a certain city, the weekly wages (in dollars) of female factory workers are normally distributed with a mean of 575 and a standard deviation of 50. Find the probability that a female factory worker selected at random from the city makes a weekly wage of \$550 to \$650. Answer : _____.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

- (10 points) The production of a certain company is given by the function $f(x, y) = 50x^{1/3}y^{2/3}$ when x units of labor and y units of capital are utilized. Find the approximate percentage change in the production of the company if labor is increased by 2% and capital is increased by 1%.
- (10 points) Find the average value of the function $f(x, y) = e^{-x^2}$ over the plane region R which is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$.
- (10 points) A tank initially contains 20 gal of pure water. Brine containing 2 lb of salt per gallon flows into the tank at a rate of 3 gal/min, and the well-stirred mixture flows out of the tank at the same rate.
 - How much salt is present in the tank at any time t ?
 - How much salt is present at the end of 20 min?
 - How much salt is present in the long run?
- (10 points) Find a function f given that (1) the slope of the tangent line to the graph of f at any point $P(x, y)$ is given by the expression $3xy$ and (2) the graph of f passes through the point $(0, 2)$.
- (10 points) The probability function f associated with a continuous random variable X has the form $f(x) = ax^2 + bx$ ($0 \leq x \leq 1$). If $E(X) = 0.6$, find the values of a and b .

(下頁還有試題)

6. (10 points) Let $f(x, y) = xy$ be the joint probability density function for the random variables X and Y on $D = \{(x, y) | 0 \leq x \leq 1; 0 \leq y \leq 2\}$.
- Find $P(X < Y)$.
 - Find $P(2X + Y \leq 1)$.

(試題結束)