管院微積分聯合教學　段考六　　Jun 19, 2012

考試時間 120 分鐘，題目卷為兩張紙，共3頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The improper integral of the form $\int_a^{\infty} e^{-px}dx$ is convergent if $p \le 0$ and divergent if $p > 0$.

2. The total differential of $w = f(x, y, z) = x\sqrt{y} + y\sqrt{z}$ is
$dw = \sqrt{y}dx + \left(\dfrac{x}{2\sqrt{y}} + \sqrt{z}\right)dy + \dfrac{y}{2\sqrt{z}}dz.$

3. If $\lim\limits_{n\to\infty} a_n = 0$, then $\sum\limits_{n=1}^{\infty} a_n$ converges.

4. If $\lim\limits_{n\to\infty} a_n b_n$ exists, then both $\lim\limits_{n\to\infty} a_n$ and $\lim\limits_{n\to\infty} b_n$ must exist.

5. $\lim\limits_{x\to 0^+} \dfrac{\cos x}{x} = \lim\limits_{x\to 0^+} \dfrac{-\sin x}{1} = 0.$

6. If $\sum\limits_{n=0}^{\infty} a_n(x-a)^n$ has radius of convergence R, then $\sum\limits_{n=0}^{\infty} {a_n}^2(x-a)^n$ has radius of convergence R^2.

7. The Taylor series of $f(x) = e^x$ at $x = 1$ is $f(x) = \sum\limits_{n=0}^{\infty} \dfrac{e}{n!}(x-1)^n$, $-\infty < x < \infty$.

8. The graph of $f(x) = \sin x + \cos x$ is concave downward on the interval $\left(0, \dfrac{\pi}{2}\right)$.

9. If $|r| > 1$, then $\sum\limits_{n=2}^{\infty} \dfrac{1}{r^n} = \dfrac{1}{r(r-1)}$.

10. Consider the function $f(x) = x^3 - 1.5x^2 - 6x + 1$. Newton's method works for solving the equation $f(x) = 0$ by choosing the initial estimate $x_0 = -1$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the volume of the solid bounded above by the surface $z = f(x, y) = \dfrac{2y}{1+x^2}$ and below by the plane region R which is bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$.

Answer：__________.

2. Determine whether the series $\sum_{n=1}^{\infty}\left[\dfrac{1}{3^n} - \dfrac{1}{n(n+1)}\right]$ converges or diverges. If it converges, find its sum. Answer：__________.

3. Find the radius of convergence of $\sum_{n=0}^{\infty}(n+1)(x+2)^n$. Answer：__________.

4. Find the third Taylor polynomial of $f(x) = \ln\sqrt{1+x}$ at $x = 0$.

Answer：__________.

5. Let $f(x) = e^{5x}\sec 8x$. Find $f'(\pi)$. Answer：__________.

6. Find $\lim_{x\to 1}\left(\dfrac{1}{\ln x} - \dfrac{1}{x-1}\right)$. Answer：__________.

7. Find $\int \sec^2 3x\sqrt{1+\tan 3x}\,dx$ Answer：__________.

8. Find the iterative formula of Newton's method for solving the equation $f(x) = x^3 - 10 = 0$. Answer：__________.

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Determine whether each series converges or diverges. Give your reason.

a. $\sum_{n=1}^{\infty}\left[\left(\dfrac{3}{4}\right)^n + \dfrac{1}{n^{4/3}}\right]$

b. $\sum_{n=1}^{\infty}\dfrac{n}{\sqrt{3n^2+5}}$

2. (*10* points) Let $f(x) = e^{-x}$. Find a bound in the error incurred in approximating $f(x)$ by the second Taylor polynomial, $P_2(x)$, of f at $x = 0$ in the interval $[0, \frac{1}{2}]$.

3. (*10* points) Use the fourth-degree Taylor polynomial to approximate

$$\int_0^{0.3}\ln(1+x^2)\,dx.$$