微積分聯合教學　段考一　**Oct 23, 2012**

考試時間 120 分鐘，題目卷爲兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. We can obtain the graph of $y = x^2 - 6x + 10$ by shifting the graph of $y = x^2$ 3 units to the right and then 1 unit upward.

2. $f(x) = \dfrac{x}{x+1}$ is an even function.

3. The graph of
$$f(x) = \begin{cases} x \sin \dfrac{\pi}{x}, & \text{if } x \neq 0. \\ 0, & \text{if } x = 0. \end{cases}$$
has a tangent line at the origin $(0, 0)$.

4. If $f(x) = x|x|$, then $f'(0)$ does not exist.

5. If $(f \circ g)(x) = (f \circ h)(x)$, then $g(x) = h(x)$.

6. If $g(x) = x^5$, then $\lim\limits_{x \to 2} \dfrac{g(x) - g(2)}{x - 2} = 80$.

7. If $|f|$ is continuous at a, so is f.

8. The derivative of a differentiable, nonzero and odd function is even.

9. Vertical asymptotes of the curve $y = \dfrac{x^3 - 1}{x^2 - 1}$ are $x = \pm 1$.

10. $\dfrac{d}{dx}|x^2 + x| = |2x + 1|$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, find the domain of the composite function $g \circ f$. Answer ：_____.

2. Calculate $\lim_{x\to 0} x\cot x$. Answer ：_____.

3. Let $f(x) = (\sin x)(\cos x)(\cos 2x)(\cos 4x)(\cos 8x)$, find $f''(0)$.

Answer ：_____.

4. If $2x \le g(x) \le x^4 - x^2 + 2$ for all x, evaluate $\lim_{x\to 1} g(x)$.

Answer ：_____.

5. Find the linearization of the function $g(x) = \sqrt[3]{1+x}$ at $x = 0$.

Answer ：_____.

6. How close to 2 do we have to take x so that $5x + 3$ is within a distance of 0.01 from 13? Answer ：_____.

7. Let $y = \cot(\cos^2 3\theta)$. Find $\dfrac{dy}{d\theta}$. Answer ：_____.

8. Find $\lim_{x\to 1}\left(\dfrac{1}{x-1} + \dfrac{1}{x^2-3x+2}\right)$. Answer ：_____.

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find $\dfrac{dy}{dx}$ of the curve $\sin(x+y) = y^2\cos x$ by using implicit differentiation.

(下頁還有試題)

2. (*10* points)

a. (5 points) Find an equation of the tangent line to the parabola $y = x^2 - 5x + 4$ that is parallel to the line $x - 3y = 5$.

b. (5 points) Find an equation of the normal line to the parabola $y = x^2 - 5x + 4$ that is parallel to the line $x - 3y = 5$.

3. (*10* points) A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s.

a. (5 points) At what rate is his distance from second base decreasing when he is halfway to first base?

b. (5 points) At what rate is his distance from third base increasing at the same moment?

4. (*10* points) Use the limit definition to prove that $\lim_{x \to 3} \dfrac{1}{x} = \dfrac{1}{3}$.

5. (*10* points) If $f(x) = x^2 - \cos^2 x$, show that there is a number c such that $f(c) = 0$. [Hint: Use Intermediate Value Theorem.]

6. (*10* points) For what value of the constant c is the function f continuous and differentiable on $(-\infty, \infty)$?

$$f(x) = \begin{cases} \dfrac{\sin 3x^2}{5x}, & \text{if } x > 0. \\ x^2 + cx, & \text{if } x \leq 0. \end{cases}$$

(試題結束)