管院微積分聯合教學　段考一　**Oct 23, 2012**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $\lim_{x\to a} f(x)$ exists, then f is defined at $x = a$.

2. $\lim_{x\to-\infty} \dfrac{2x^2-3x+1}{x^2+2x+4} = \lim_{x\to+\infty} \dfrac{2x^2-3x+1}{x^2+2x+4}$.

3. If f is a continuous function on $[a, b]$, then there is at least one number c in [a,b] such that $f(c) = \dfrac{1}{3}(f(a) + 2f(b))$.

4. If f^2 is differentiable at a, then f is also differentiable at a.

5. If f is differentiable, then $\dfrac{d}{dx}[\dfrac{f(x)}{x^{10}}] = \dfrac{f'(x)}{10x^9}$.

6. Let $h(x) = f(g(x))$, then $h''(x) = f''(g(x))g''(x)$.

7. If f is continuous at $x = a$, then f is differentiable at $x = a$.

8. If $f(x) = ax + c$, then $\dfrac{d}{dx}f(f(x)) = \dfrac{d}{dx}f(x) \cdot \dfrac{d}{dx}f(x)$.

9. If $b^2 - 4a < 0$, then $ax^2 + bx + 1 = 0$ has no real roots.

10. Let $f(x) = \begin{cases} \dfrac{3}{2}x^2 - 1, & \text{if } -2 \le x < 1 \\ x^3, & \text{if } 1 \le x \le 2 \end{cases}$, then f is not differentiable at 1.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let $f(x) = \dfrac{\sqrt{x+1}}{x-1}$, the domain of f is $\boxed{\text{A}}$ and $f'(2) = \boxed{\text{B}}$.

Answer：$\boxed{\text{A}}$ $\boxed{\text{B}}$.

2. If $h(x) = f(2x)$, $f^{(n)}(2x) \neq 0$, then find $\dfrac{h^{(n)}(x)}{f^{(n)}(2x)}$.

Answer：_____.

3. Let $h(x) = \dfrac{f(x)g(x)}{f(x) - g(x)}$, $f(1) = 2$, $f'(1) = -1$, $g(1) = -2$ and $g'(1) = 3$.
Find $h'(1)$.

Answer：_____.

4. If $\sqrt{x+y} = x^3$, then find $\dfrac{dy}{dx}$.

Answer：_____.

5. Let $s(t) = \left(\dfrac{t}{2t+1}\right)^{5/3}$, then find $\dfrac{ds}{dt}$.

Answer：_____.

6. Let $f(x) = \begin{cases} x+2, & \text{if } x \leq 1 \\ kx^2, & \text{if } x > 1 \end{cases}$, then f will be continuous on $(-\infty, \infty)$.
Find k.

Answer：_____.

7. $\lim\limits_{x \to -2} \dfrac{2(8 - x^3)}{2x^2 - x^3}$.

Answer：_____.

8. Find the equation of the line that passes through the point $(2, 4)$ and is perpendicular the line $3x + 4y - 22 = 0$.

Answer：_____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Express $\lim_{h\to 0}\dfrac{3(2+h)^2-(2+h)-10}{h}$ as $\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$.
If $f(a)=10$, find a,$f(a+h)$, $f(x)$, $f'(x)$ and the limit $\lim_{h\to 0}\dfrac{3(2+h)^2-(2+h)-10}{h}$.

2. (*10* points) The fee charged per car in a downtown parking lot is \$100 for the first hour and \$50 for each additional half hour or part thereof, subject to a maximum of \$500. Derive a function f relating the parking fee to the length of time a car is left in the lot. Sketch the graph of f and determine the values of x for which the function f is discontinuous.

3. (*10* points) Let $y^2-xy=8$. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

4. (*10* points) The volume of a right-circular cylinder of radius r and height h is $V=\pi r^2h$. Suppose the radius and height of the cylinder are changing with respect to time t. At a certain instant of time, the radius and height of the cylinder are 10 and 2 in, and are increasing at the rate of 0.5 and 0.2 in/sec, respectively. How fast is the volume of the cylinder increasing?

5. (*10* points) Let $f(x)=\dfrac{1}{\sqrt{x^3+1}}$ and $g(x)=\dfrac{1}{\sqrt{x+1}}$.

a. Find the differentials of $f(x)$ and $g(x)$, respectively.

b. Use df and dg to approximate $\dfrac{1}{\sqrt{1.001}}$

6. (*10* points) The demand function for a certain make of exercise bicycle sold exclusively through cable television is $p=\sqrt{9-0.02x}$ $\quad(0\le x\le 450)$ where p is the unit price in hundreds of dollars and x is the quantity demanded/week. Compute the elasticity of demand and determine the range of prices corresponding to inelastic, unitary, and elastic demand. Hint: Solve the equation $E(p)=1$.

(試題結束)