

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If f' is continuous on the interval $[1, 3]$, then $\int_1^3 f'(x) dx = f(3) - f(1)$.
2. If $P(c, f(c))$ is an inflection point of the curve $y = f(x)$, then $f''(c) = 0$.
3. If f has discontinuity at 1, then $\int_0^2 f(x) dx$ does not exist.
4. All continuous functions have antiderivatives.
5. If f is continuous on $[0, 1]$, then $\int_0^1 f(x) dx = \int_0^1 f(1-x) dx$.
6. If the line $x = 1$ is a vertical asymptote of the curve $y = f(x)$, then $f(x)$ is not well-defined at $x = 1$.
7. $|\sin a - \sin b| \leq |a - b|$, for all a and b .
8. $\int_{-2}^1 \frac{1}{x^4} dx = -\frac{3}{8}$.
9. If f is periodic, then f' is periodic.
10. If $f'(x)$ exists and is nonzero for all x , then $f(1) \neq f(0)$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the limit $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x)$. Answer : _____.

2. Evaluate $\int_{-\pi/4}^{\pi/4} (x^3 + x^4 \tan x) dx$.
Answer : _____.

3. Find the limit $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \dots + \frac{1}{\sqrt{n^2 + n}} \right)$.
Answer : _____.

4. Evaluate $\int_0^{\pi/4} \frac{1 + \cos^2 \theta}{\cos^2 \theta} d\theta$.
Answer : _____.

5. Find all vertical asymptotes of the curve $y = \frac{\sin x}{x^2 - x}$.
Answer : _____.

6. Let $f(x) = x^{1/4}(2 - x)^{3/4}$. Find the absolute maximum value of $f(x)$ on $[0, 1]$.
Answer : _____.

7. Find the average value of $f(x) = \cos^4 x \sin x$ on $[0, \pi]$.
Answer : _____.

8. Find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \pi/2$. Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find $\frac{d^2}{dx^2} \int_0^x \int_1^{\sin t} \sqrt{1+u^4} du dt$.

2. (10 points) If f is continuous and $\int_1^3 f(x) dx = 8$. Show that f takes on the value 4 at least once on the interval $[1, 3]$.

3. (10 points) Find the dimension of the isosceles(等腰) triangle of largest area that can be inscribed in a circle of radius r .

4. (10 points) Find the volume of the solid obtained by rotating about the y -axis the region between $y = x$ and $y = x^2$.

5. (10 points)

a. If a and b are positive numbers, show that

$$\int_0^1 x^a(1-x)^b dx = \int_0^1 x^b(1-x)^a dx.$$

(Hint: Use change of variable, let $u = 1 - x$.)

b. Evaluate $\int_{-1}^2 (x - 2|x|) dx$.

6. (10 points) Let $f(x) = \frac{x^3}{x^2 + 1}$. Find **a.** its domain, **b.** critical numbers, **c.** intervals of increasing/decreasing, **d.** local maximum/minimum values, **e.** concavity, **f.** inflection points, **g.** slant, horizontal and vertical asymptotes. **h.** Then sketch the graph of $f(x)$.

(試題結束)