

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $f'(c) = 0$, then f has a relative extrema at $x = c$.
2. Any rational function has at most one horizontal asymptote.
3. If f is continuous on $[a, b]$, then f has both an absolute maximum value and an absolute minimum value on $[a, b]$.
4. If $0 < b < 1$ and $x > y$, then $b^x > b^y$.
5. $\lim_{m \rightarrow \infty} \left(1 + \frac{1}{m}\right)^{-m} = e^{-1}$.
6. Let $A, B > 0$, $A = B(2)^{3t}$, then $t = \frac{1}{3} \ln \frac{A}{B}$.
7. The function $f(x) = \frac{1}{\ln x}$ is continuous on $[1, \infty)$.
8. Let r be the nominal rate, r_{eff} be the effective rate, if the interest is compounded m times half a year, then $P(1 + r_{\text{eff}}) = P \left(1 + \frac{r}{2m}\right)^{2m}$.
9. $\frac{d}{dx} e^{f(g(x))} = e^{f(g(x))} \cdot f'(g(x))$.
10. $\frac{d}{dx} (\ln 2x) \neq \frac{d}{dx} \left(\ln \frac{x}{3}\right)$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let a and b be the absolute maximum and absolute minimum values of the function

$$f(x) = \frac{x}{x^2 + 1}, \text{ find } a + b.$$

Answer : _____.

2. If $yx^2 + e^y = 10$, find y' .

Answer : _____.

3. Let $\frac{A}{1 + Be^{t/2}} = C$. Find t if $A \neq C$, $B \neq 0$ and $C \neq 0$.

Answer : _____.

4. Let $f(x) = 2 + \frac{5}{(x-2)(\ln x + 1)}$. Find all vertical asymptotes and horizontal asymptotes of $f(x)$.

Answer : _____.

5. Let $g(t) = t\sqrt{t^2 + 1}$ on $[-3, 2]$. Find the absolute maximum value of $g(t)$.

Answer : _____.

6. Let $f(x) = a + b \ln x$, if $f(1) = 2$, $f(2) = 4$, find $f(8)$.

Answer : _____.

7. Let $y = \frac{\sqrt{4 + 3x^2}}{3\sqrt{x^2 + 1}}$. Find $\frac{dy}{dx}|_{x=1}$.

Answer : _____.

8. Let $y = x^{\ln x}$. Find $\frac{dy}{dx}$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) The unit price of a certain commodity is given by $P = f(t) = 6 + 4e^{-2t}$, $t \in [0, \infty)$, where P is measured in dollars and t is measured in months.
 - a. Find the interval where f is increasing and the interval where f is decreasing.
 - b. Find all possible absolute extrema.
 - c. Find the interval where f is concave downward and the interval where f is concave upward.
 - d. Find inflection points if any.
 - e. Find all vertical asymptotes and horizontal asymptotes if any.
 - f. Sketch the graph of f .

2. (10 points) The unit selling price p (in dollars) and the quantity demanded x (in pairs) of a certain brand of women's gloves is given by the demand equation

$$P = 100e^{-0.0001x}, \quad 0 \leq x \leq 20000$$

- a. Find the revenue function R .
 - b. Find the marginal revenue function.
 - c. What is the marginal revenue when $x = 10000$?
 - d. How many pairs of the gloves must be sold to yield a maximum revenue ?
 - e. What will be the maximum revenue ?
3. (10 points) Find the present value of \$59673 due in 5 years at an interest rate of 8%.
 - a. Compounded monthly.
 - b. Compounded daily.
 - c. Compounded continuously.
 4. (10 points) Carbon 14, a radioactive isotope of carbon, has a half-life of 5770 years. Skeletal remains of the so-called Pittsburgh Man, unearthed in Pennsylvania, had lost

82% of the C-14 they originally contained. Determine the approximate age of the bones.

5. (10 points) Postal regulations specify that a parcel sent by priority mail may have a combined length and girth of no more than 108 in. Find the dimensions of a rectangular package that has a square cross section and the largest volume that may be sent via priority mail. What is the volume of such a package?
6. (10 points) The graph of the derivative $y = f'(x)$ is given in the following Figure. Given $A(0, -1.3)$, $B(0.3, 0)$, $C(1, 1)$, $D(1.4, 0)$, $E(2.2, -2)$, $F(3, 0)$, $G(5, 1.5)$.
- Find the open intervals where f is increasing and the open intervals where f is decreasing.
 - Find all possible values of x where f has relative extrema.
 - Find the open intervals where f is concave downward and the open intervals where f is concave upward.
 - Find all possible values of x where $(x, f(x))$ are inflection points.

(試題結束)