

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If  $f$  has discontinuity at 1, then  $\int_0^2 f(x) dx$  does not exist.
2. If  $f$  is periodic, then  $f'$  is periodic.
3. If  $f$  is continuous on  $[0, \infty)$  and  $\int_1^{\infty} f(x) dx$  is convergent, then  $\int_0^{\infty} f(x) dx$  is convergent.
4.  $\frac{d}{dx}(10^x) = x10^{x-1}$ .
5.  $\lim_{x \rightarrow \pi^-} \frac{\tan x}{1 - \cos x} = \lim_{x \rightarrow \pi^-} \frac{\sec^2 x}{\sin x} = \infty$ .
6.  $\sin^{-1}(-1) = \frac{3}{2}\pi$ .
7. Let  $f(x) = \begin{cases} e^{-1/x^2}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$  Then  $f$  is differentiable on  $(-\infty, \infty)$ .
8. For any polynomial function  $P(x)$ , we have  $\lim_{x \rightarrow \infty} \frac{P(x)}{e^x} = 0$ .
9.  $\int_0^4 \frac{x}{x^2 - 1} dx = \frac{1}{2} \ln 15$ .
10.  $\int_1^{\infty} \frac{1}{x\sqrt{2}} dx$  is convergent.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the volume of the solid obtained by rotating about the  $y$ -axis the region between  $y = x$  and  $y = x^2$ .

Answer : \_\_\_\_\_.

2. Find  $\lim_{x \rightarrow 0^+} (1 + 2 \sin x)^{\frac{1}{x}}$ .

Answer : \_\_\_\_\_.

3. Evaluate  $\int \sin 4x \cos 5x \, dx$ .

Answer : \_\_\_\_\_.

4. The arc of the parabola  $y = x^2$  from  $(1, 1)$  to  $(2, 4)$  is rotated about the  $y$ -axis. Find the area of the resulting surface.

Answer : \_\_\_\_\_.

5. Evaluate  $\int \sqrt{\frac{1-x}{1+x}} \, dx$ .

Answer : \_\_\_\_\_.

6. Find the length of the curve  $y = \int_1^x \sqrt{t^3 - 1} \, dt$ ,  $1 \leq x \leq 4$ .

Answer : \_\_\_\_\_.

7. Evaluate  $\int_0^1 \ln x \, dx$ .

Answer : \_\_\_\_\_.

8. If  $f(x) = \ln x + \tan^{-1} x$ , find  $(f^{-1})' \left( \frac{\pi}{4} \right)$ .

Answer : \_\_\_\_\_.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Let  $y = \cot(\cos^2 3\theta)$ . Find  $\frac{dy}{d\theta}$ .

2. (10 points) Find the integral.

a.  $\int \sec^3 \theta \, d\theta$

b.  $\int_0^1 \sqrt{1+4x^2} \, dx$

3. (10 points) Use logarithmic differentiation to find

$$\frac{d}{dx} \sqrt[5]{\frac{x(x+1)(x-2)}{(x^2+1)(3x+2)}}.$$

4. (10 points) Determine whether

$$\int_1^{\infty} \frac{x+1}{\sqrt{x^4-x}} \, dx$$

is convergent or divergent.

5. (10 points) Evaluate the integral.

a.  $\int \frac{\tan^3 x}{\cos^3 x} \, dx$

b.  $\int \frac{4x^2 - 3x + 2}{4x^2 - 4x + 3} \, dx$

6. (10 points) If  $f$  is continuous function such that

$$\int_0^x f(t) \, dt = xe^{2x} + \int_0^x e^{-t} f(t) \, dt$$

for all  $x$ , find an explicit formula for  $f(x)$ .

(試題結束)