

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If f' is continuous on $[0, 1]$ and if the graph of f passes through the points $(0, 1)$, $(0.5, 5)$ and $(1, 5)$, then $\int_0^1 f'(x)dx = 5$.
2. The substitution $u = \sqrt{1 + \sqrt{x}}$ transforms $\int \sqrt{1 + \sqrt{x}}dx$ into $4 \int (u^4 - u^2)du$.
3. If f is a continuous and decreasing function on $[0, 2]$, then $\int_0^1 f(x)dx \geq \int_1^2 f(x)dx$.
4. $\int \frac{1}{(x+1)^2} dx = \ln |(x+1)^2| + C$.
5. The producers' surplus is given by

$$PS = \int_0^{\bar{x}} S(x)dx - \bar{p}\bar{x}$$

where $S(x)$ is the supply function, \bar{p} is the unit market price, and \bar{x} is the quantity supplied.

6. $\int g'(x) \ln x dx = g(x) \ln x - \int \frac{g(x)}{x} dx$.
7. If f is continuous on $[a, b]$ and $a < c < b$, then

$$\int_b^c f(x)dx = \int_a^c f(x)dx - \int_a^b f(x)dx$$

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8. If f and g are continuous on $[a, b]$ and $f(t) \geq g(t)$ for all t in $[a, b]$,
then $\int_a^b [f(t) - g(t)] dt \geq 0$.
9. Given integral formula $\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$,
then $\int_0^2 \frac{dx}{(5 - x^2)^{3/2}} = \frac{x}{25 \sqrt{25 - x^2}} \Big|_0^2$.
10. If f is continuous on $[a, b]$, then $\int_a^b f(x) dx$ is equal to the area of the region above $[a, b]$ minus the area of the region below $[a, b]$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find $\int \sqrt[3]{x} \ln x dx$.

Answer : _____.

2. An investor is presented with a choice of two investments: an established clothing store and a new computer store. Each choice requires the same initial investment and each produces a continuous income stream of 4%, compounded continuously. The rate of flow of income from the clothing store is $f(t) = 12,000$ and the rate of flow of income from the computer store is expected to be $g(t) = 10,000e^{0.05t}$. Compute the future values of these investments and determine which is the better choice over the next 5 years. (Note: $e^{0.25} \approx 1.28$, $e^{0.2} \approx 1.22$)

Answer : A B C.

3. Given the equation $\sqrt{x+y} = x^3 \ln y$, find $\frac{dy}{dx}$.

Answer : _____.

4. A certain city's rate of electricity consumption is expected to grow exponentially with a growth constant of $k = 0.04$. If the present rate of consumption is 40 million-kilowatt-hours (kWh) per year, what should be the total production of electricity over the next 3 years in order to meet the projected demand?

Answer : _____.

5. Let $y = x^{\ln x}$. Find $\frac{dy}{dx}$.

Answer : _____.

6. Find the area of the region bounded by the graph of the function $f(x) = x\sqrt{1-x^2}$ and the x-axis.

Answer : _____.

7. A state lottery commission pays the winner of the “Million Dollar” lottery 10 annual installments of \$40,000 each. If the prevailing interest rate is 2%/year compounded continuously, find the present value of the winning ticket.

(Note: $e^{-0.2} \approx 0.82$)

Answer : _____.

8. Suppose f'' is continuous on $[0, 1]$, $f(1) = 2$, $f'(1) = 2$ and the average value of f on $[0, 1]$ is 2. Evaluate $\int_0^1 x^2 f''(x) dx$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Given the demand function $D(x) = (x - 5)^2$ and the supply function $S(x) = x^2 + x + 3$, find each of the following.
 - a. The equilibrium point.
 - b. The consumers' surplus at the equilibrium point.
 - c. The producers' surplus at the equilibrium point.

2. (10 points) The production of oil (in millions of barrels per day) extracted from oil sands in Canada is projected to grow according to the function

$$P(t) = \frac{5}{1 + 4e^{-0.2t}} \quad (0 \leq t \leq 12)$$

where t is measured in years, with $t = 0$ corresponding to 2005. What is the expected total production of oil from oil sands over the years from 2005 until 2017 ($t = 12$)?

3. (10 points) Evaluate the following integrals.

- a. $\int_0^4 \frac{x}{\sqrt{2x+1}} dx$

- b. $\int e^{2x} \sqrt{1 + 3e^x} dx$

4. (10 points) In a study on the effects of World War II on the U.S. economy, an economist used data from the U.S. Census Bureau to produce the following Lorentz curves for the distribution of U.S. income in 1935 and in 1947:

$$f(x) = x^{2.4} \text{ (Lorentz curve for 1935)}$$

$$g(x) = x^{1.6} \text{ (Lorentz curve for 1947)}$$

Compute the coefficient of inequality for each Lorentz curve.

5. (10 points) Steve Nash is an excellent NBA player who has just become a free agent(自由球員). His agent(經紀人), Bill Duffy, begins negotiations with an interested team by asking for a contract that provides Steve with an income stream given by $R(t) = 800,000 + 340,000t$ over 10 years, where t is in years. Steve signs a contract for the income stream as his agent asks but decides to live on \$500,000 each year, investing the rest at 8%, compounded continuously. What is the accumulated future value of the remaining income, assuming an interest rate of 8%, compounded continuously for the next 10 years ? (Note: $e^{0.8} \approx 2.23$)
6. (10 points) The unit selling price p (in dollars) and the quantity demanded x (in pairs) of a certain brand of women's gloves is given by the demand equation

$$p = 100e^{-0.0001x}, \quad 0 \leq x \leq 20000$$

- a. Find the revenue function R .
- b. Find the marginal revenue function.
- c. What is the marginal revenue when $x = 10000$?
- d. How many pairs of the gloves must be sold to yield a maximum revenue ?
- e. What will be the maximum revenue ?

(試題結束)